The Analytical Formulations for Vehicle Motion Planning

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Abstract:

We are now just entering the age of intelligent electro-mechanical systems. In this case, a multi-wheeled vehicle is to be modernized by opening up the architecture (it is fully modular and can be assembled on demand). In particular, the mobile platform can have any body geometry for N wheels (supported by articulated suspensions of 2 to 3 DOF), all acting in parallel. Given the motion specifications of each wheel/suspension leads to a programming and mathematical nightmare which rapidly loses all physical meaning. Here, we decide to specify the motion of the platform up to, say, the 4th order, algebraically compute the required input parameters at each of the N wheels (up to the 4th order) and then evaluate the actuator and traction resources to see if those input commands are satisfied and with what margins (positive means success, negative means failure). The ultimate goal is to balance all the margins in real time to ensure system success and how marginal that success is even in poor weather, rough terrain, emergency maneuvers, etc. This, then, leads to a 5 to10 milli-sec. decision making problem which must be made in terms of physically meaningful criteria, which is the basis for the formulation presented in this paper

Key Words: Vehicle Planning Motion, Instant Center.

1. Introduction

This summary intends to integrate what has been accomplished in a major report at
The University of Texas on mobile platforms [Kulkarni 2009] in order to create a balanced development for intelligent vehicles. That work concentrated on the theoretical understanding and physical meaning of a parametric formulation for planar motion mobile platforms. To do so, we had to anchor this study on the location of the instant centers up to the 5th order. These instant centers begin to provide physical meaning for the numerical specification of the motion. For example, if the velocity instant center I_1 is fixed, then the motion is pure rotation. When $I_1 \Rightarrow \infty$, then the motion is pure translation. Should the second order instant center I_2 for acceleration also have these properties, then the motion is instantaneously in a pure rotation or pure translation, etc. The future goal is to acquire and coalesce this physical meaning so the operator knows how to specify the motion in order to synthesize a motion plan that meets the operator's real motion needs. Today, vehicles are operated much like one would operate a bicycle. Point it to where you want to go and then decide if you are getting there. This is not modern control where the system augments the human's command. Also, few vehicles operate under ideal conditions. They may operate on a muddy slope, on ice, in sand, in rough terrain, etc. These conditions demand too much of the operator, resulting in decisions that over commit the vehicle beyond its performance and safety limits.

Here, once the motion has been synthesized, we show how to analytically determine what the input commands are for each and every wheel subsystem. These wheel subsystems may or may not be capable of responding to these commands, which leads to another level of decision making. Once the wheel subsystem has evaluated its capacity to respond, that condition is reported to the system level to see how well the resulting motion of the platform will satisfy the desired (specified) motion. This, then, leads to further evaluation (and decision making) to readjust the motion specification to meet the performance realities.
All of this must occur in milli-sec. In the past, the mathematics of just the description of the motion was mired in an implicit and uncertain computation. Here, we show that all forward commands (position, velocity, acceleration, force, torque, etc.) can be calculated in parallel for each of the active wheel

subsystems. The inverse computations can also be obtained (in the small) in parallel without any uncertainty. There is no suggestion here that all the development is done. We only suggest that, here, we have broken the implicit computational log jam (in the literature) in favor of a direct decision making structure based on explicit computations, with no mathematical uncertainty (such as pseudo inverses). This, then, allows the growth of real science to accelerate the concentration on the real operational issues:

1. How to accurately specify and synthesize the motion that has physical meaning to the operator?
2. How to establish the performance limits of each active wheel subsystem relative to the actual contact surface condition?
3. How to evaluate whether the motion adequately meets the desired motion with certain performance and safety criteria?
4. How to extend this process to mission planning to evaluate if sufficient resources are available to meet a long duration operation?
5. How to couple the operator and system intelligence into a high level decision making process in what we now call extended autonomy?

2. Motion Planning Literature Review And Summary

The principal literature is given in Table 2. A key reference paper is that by [Muir 1987] where they give an extensive listing of the large variability of mobile platform configurations (see Fig. 1, 2). Unfortunately, this was followed by a beautifully constructed analytical formulation of the motion of this generalized architecture by [Campion 1996] that led to a completely implicit (and unnecessary) algebraic computation with uncertain results. [Yi 2002] built on these results. The uncertainty arises in that the mathematical framework of Campion chooses the wheel input parameters as deterministic (not the desired output parameters of the platform). This, then, leads to many more input parameters (easily 3 to 10x more) than there are output parameters; hence, the need for the implicit computation based on a pseudo inverse. Here we show that this implicit inverse computation can be "inverted" to a parallel explicit forward computation where the desired motion of the platform (i.e., it is synthesized in the format of the kinematics which was founded in 1875) can be used to directly compute the input commands of the active wheels (without mathematical uncertainty).
To do so means that we must build on the well-established kinematics literature of Bottema, Veldkamp, Tesar, and others. Also, considerable work on motion platform dynamics has been achieved by Freeman and Tesar, Holmberg and Khatib, Wong and others.

Figure 1

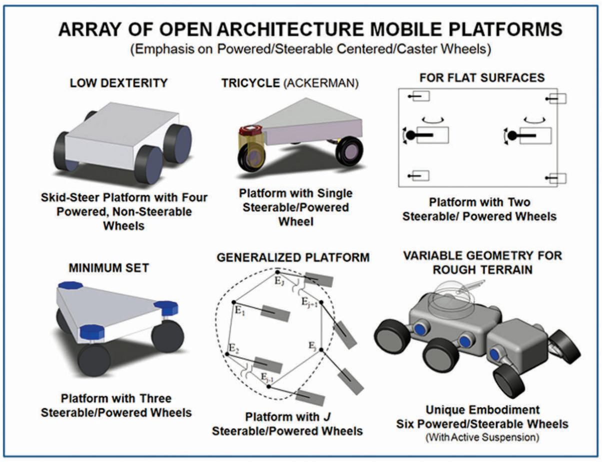


Figure 2

Table 1: Literature Review		
Instantaneous Invariants Theory	Bottema, 1961; Bottema and Roth, 1979	• Introduced the theory of instantaneous invariants • Presented an algebraic formulation to describe planar and spatial rigid body motion using instantaneous invariants
	Tesar et al. 1967 - 2009	• generalized the instantaneous formulation for kinematic motion synthesis in terms of multiply separated positions, 3 D end-effector motion, platform motion, etc.
	Cowie, 1961	• Vector based formulation for the first and second order IC with physically relevant discussions
Spatial Case Study	Veldkamp, 1969	• Studied the acceleration center and acceleration field of the rigid body spatial motion with a study of special cases
	Ridley, 1992	• Used screw theory and its time derivative to describe the spatial motion of a rigid body for up to the second order
Mobile Platform Kinematics	Muir and Newman, 1989	• Presented a general approach to model mobile platforms on the lines similar to manipulators (Thomas & Tesar, 1982).
	Campion et al., 1996	• Presented an implicit method for kinematic modeling of mobile platforms using kinematic constraints on various wheel configurations
	Yi and Kim, 2002	• Presented inverse kinematics methodology for redundantly actuated mobile platform based on Campion
Mobile Platform Dynamics	Freeman and Tesar, 1988	• Proposed a generalized dynamic modeling methodology for serial and parallel robotic systems
	Holmberg and Khatib, 2000	• Presented Newton-Euler based dynamic model for mobile platform with caster wheels
	Wong, 2001	• Detailed study of vehicle dynamics with an emphasis on wheel-ground interaction properties

3. Instant Center Formulation

In Figure 3, we show the parametric description of the planar motion (X, Y, θ) of a rigid body described by body fixed coordinates (x, y) for any point E (x_E ,y_E) in the body, located at X_E ,Y_E in the reference coordinate system. The point of interest (POI) or P (x_p ,y_p) is located at (X_p ,Y_p) in the reference system. Then,

$$\begin{aligned} X_E &= X_P + x_E \cos\theta - y_E \sin\theta \\ Y_E &= Y_P + x_E \sin\theta + y_E \cos\theta \end{aligned} \tag{1}$$

Given the motion specification ($\dot{X}_P, \dot{Y}_P, \dot{\theta} = \omega$), then Eq. (1) can be differentiated to give:

$$\begin{aligned} \dot{X}_E &= \dot{X}_P - x_E \sin\theta\omega - y_E \cos\theta\omega \\ \dot{Y}_E &= \dot{Y}_P + x_E \cos\theta\omega - y_E \sin\theta\omega \end{aligned} \tag{2}$$

Now, setting $\dot{X}_E, \dot{Y}_E = O$ in Eq. (2) results in a linear set of two equations in the unknowns x_E ,y_E for the location of the first instant Center I_l in the moving body and I_l (X_{I1} , Y_{I1}) in the reference body. Clearly, if we differentiate again, given the additional motion specifications ($\ddot{X}_p, \ddot{Y}_p, \ddot{\theta} = \alpha$), then the result is

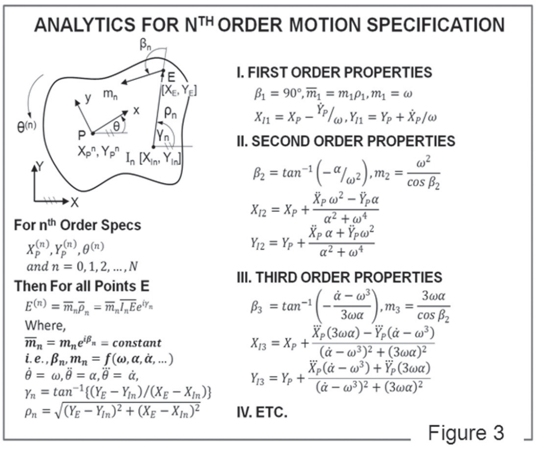


Figure 3

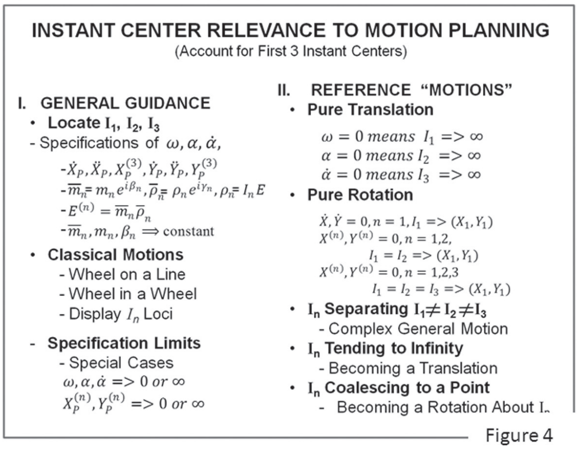


Figure 4

another set of two linear equations in the coordinates of $I_2(X_{I2}, Y_{I2})$ in the reference system. As shown in Figure 3, this process can be generalized to any order of the specified motion to give the location I_n (X_{In} , Y_{In}) of the n^{th} order instant center. This simple process yields a remarkable level of physical meaning to these higher order specifications (which is essential for motion planning by the operator). For example, given X_E ,Y_E, and X_{In} , Y_{In}, the distance $I_nE = \rho_n$ is a key concept in the motion. The total vector motion $E^{(n)}$ of point E is now given by $E^{(n)} = m_n e^{i(\beta_n + \alpha_n)}$ where the magnitude m_n is proportional to the magnitude of ρ_n and the direction of $E^{(n)}$ is given by $\beta_n + \gamma_n$. Here, β_n is the vector direction of ρ_n where β_n exhibits the remarkable property of being constant for all points E in the moving system.
These parameters are listed up to the third order in Figure 3.

4. General Discussion on Instant Centers

Here, we summarize the first and second order motion parameter choices that the IC based formulation offers for composing a motion plan of a mobile platform. We restrict ourselves to the motion of the POI on the body for this discussion. See Fig. 4 and Table 4.

First Order Motion

1. If we choose the velocity of the POI as v_P, it fixes the orientation of ρ_1 (since $\beta_1 = 90$)
   a. we can choose the radius ρ_1 to fix the angular velocity ω of the body and also the first order IC, I_1.
   b. or,we can choose the angular velocity ω of the body to fix the radius ρ_1 and also the first order IC, I_1.
2. If we choose the radius ρ_1 of the POI, it fixes the orientation of v_p (since $\beta_1 = 90$)
   a. we can choose v_P to fix the angular velocity ω of the body and also the first order IC, I_1.
   b. or, we can choose the angular velocity ω of the body to fix the velocity v_P and also the velocity IC, I_1.

Second Order Motion

We know ω from the first order motion computation.

1. The normal component of the total acceleration a_p^n of the POI fixes the location of I_2
    a. choose the angular acceleration α to fix the tangential component a_p^t
    b. or, choose the tangential component a_p^t to fix the angular acceleration α
2. The tangential component of acceleration a_p^t fixes the direction of the radius ρ_2)
    a. choose the angular acceleration α to fix the location of I_2. Since angular acceleration is already known, we can compute the normal component of the total acceleration a_p^n
    b. choose the radius ρ_2 to fix the location of I_2. since angular acceleration is already known, we can compute the normal component a_p of the total acceleration

Table 4: Special Case Scenarios for the First and Second Order ICs

Condition	Result/Consequence
$I_1 => \infty$	$\omega = 0, \alpha = 0$: Stationary Translation
	$\omega = 0, \alpha \neq 0$: Instantaneous Translation
$I_1 \equiv I_2$	$\omega \neq 0, \alpha \neq 0$ Instant Centers Coincident; Pure Rotation
$I_1 => I_2$	Going Towards a Condition of Pure Rotation, I_1 is stationary
$I_1 \neq> I_2$	I_2 is Going Away from I_1 to Give a More Complex Motion
I_2 is Stationary	Accelerating Around a Point Acting as the Acceleration Center

Table 2: Summary of the IC Based Kinematic Formulation for Mobile Platforms

	IC Location	The Orientation Angle	Times States of a General Point 'E'
First Order	$X_{I1} = X_P - \frac{\dot{Y}_P}{\omega}$ $Y_{I1} = Y_P + \frac{\dot{X}_P}{\omega}$	$\beta_1 = 90^o$	$\dot{X}_E = -\omega . Y_{\rho 1}$ $\dot{Y}_E = \omega . X_{\rho 1}$ $m_1 = \rho_1 . \omega,\ \beta_1 = 90^o$
Second Order	$X_{I2} = X_P + \frac{\ddot{X}_P \omega^2 - \ddot{Y}_P \alpha}{\alpha^2 + \omega^4}$ $Y_{I2} = Y_P + \frac{\ddot{X}_P \alpha + \ddot{Y}_P \omega^2}{\alpha^2 + \omega^4}$	$\tan \beta_2 = -\frac{\alpha}{\omega^2}$	$\ddot{X}_E = -\omega^2 [X_{\rho 2} - Y_{\rho 2} tan\beta_2]$ $\ddot{Y}_E = -\omega^2 [X_{\rho 2} . tan\beta_2 + Y_{\rho 2}]$ $m_2 = \left\| -\frac{\omega^2}{cos\beta_2} \rho_2 \right\| e^{j(\gamma_2 + \beta_2)}$
Third Order	$X_{I3} = X_P + \frac{X_P^{(3)}(3\omega\alpha) - Y_P^{(3)}(\dot{\alpha} - \omega}{(\dot{\alpha} - \omega^3)^2 + (3\omega\alpha)^2}$ $X_{I3} = Y_P + \frac{X_P^{(3)}(\dot{\alpha} - \omega^3) + Y_P^{(3)}(3\omega}{(\dot{\alpha} - \omega^3)^2 + (3\omega\alpha)^2}$	$\tan \beta_3 = -\frac{\dot{\alpha} - \omega^3}{3\omega\alpha}$	$X_E^{(3)} = -3\omega\alpha[X_{\rho 3} - Y_{\rho 2} tan\beta_3$ $Y_E^{(3)} = -3\omega\alpha[X_{\rho 3} tan\beta_k + Y_{\rho 3}$ $m_3 = \left\| -\frac{3\omega\alpha}{cos\beta_3} \rho_3 \right\| e^{j(\gamma_3 + \beta_3)}$

Fourth Order	$X_{I4} = X_P + \dfrac{X_P^{(4)}(4\omega\dot{\alpha}+3\alpha^2-\omega^4) - Y_P^{(4)}(\ddot{\alpha}-6}{(\ddot{\alpha}-6\omega^2\alpha)^2+(4\omega\dot{\alpha}+3\alpha^2-\omega^4)}$ $Y_{I4} = Y_P + \dfrac{X_P^{(4)}(\ddot{\alpha}-6\omega^2\alpha) + Y_P^{(4)}(4\omega\dot{\alpha}+3\alpha^2 -}{(\ddot{\alpha}-6\omega^2\alpha)^2+(4\omega\dot{\alpha}+3\alpha^2-\omega^4)}$	$\tan\beta_4 = -\dfrac{\ddot{\alpha}-6\omega^2\alpha}{4\omega\dot{\alpha}+3\alpha^2-\omega^4}$	$X_E^{(4)} = -(4\omega\dot{\alpha}+3\alpha^2-\omega^4).[X_{\rho4}-Y_{\rho4}\tan\beta_4]$ $Y_E^{(4)} = -(4\omega\dot{\alpha}+3\alpha^2-\omega^4).[X_{\rho4}\tan\beta_4+Y_{\rho4}]$ $m_4 = \left\lvert -\dfrac{4\omega\dot{\alpha}+3\alpha^2-\omega^4}{\cos\beta_4}\rho_4\right\rvert e^{j(\gamma_4+}$
Fifth Order	$X_{I5} = X_P + \dfrac{X_P^{(5)}(10\alpha\dot{\alpha}+3\omega\ddot{\alpha}-10\omega^3\alpha) - Y_P^{(5)}(}{(\alpha^{(3)}-10\omega^2\dot{\alpha}-15\omega\alpha^2+\omega^5)^2+}$ $Y_{I5} = Y_P + \dfrac{X_P^{(5)}(\alpha^{(3)}-10\omega^2\dot{\alpha}-15\omega\alpha^2+\omega^5)+}{(\alpha^{(3)}-10\omega^2\dot{\alpha}-15\omega\alpha^2+\omega^5)^2+}$	$\tan\beta_5 = -\dfrac{\alpha^{(3)}-10\omega^2\dot{\alpha}-1}{10\alpha\dot{\alpha}+3\omega\ddot{\alpha}}$	$X_E^{(5)} = -(10\alpha\dot{\alpha}+3\omega\ddot{\alpha}-10$ $[X_{\rho5}-Y_{\rho5}\tan\beta_5]$ $Y_E^{(5)} = -(10\alpha\dot{\alpha}+3\omega\ddot{\alpha}-10$ $[X_{\rho5}\tan\beta_5+Y_{\rho5}]$ $m_5 = \left\lvert -\dfrac{10\alpha\dot{\alpha}+3\omega\ddot{\alpha}-10\omega}{\cos\beta_5}\right.$

5. Numerical Example of Motion Planning

Consider a mobile platform traversing a trajectory that changes from concave to convex at point C so as to make an 'S' shaped curve as shown in Fig. 5. In this case, the body is always aligned with the direction of travel, such that all the ICs for the velocity, acceleration, jerk, etc., are located at the center of the curvature. When the mobile platform crosses point C, the normal acceleration, jerk etc. instantaneously switch to the opposite direction resulting in shock and motion uncertainty.

Using IC based motion programming; we can remove this crossover shock and uncertainty with a dexterous platform as follows. To prevent the shock, we put a restriction on the motion whereby point C becomes a stationary inflection point. To accomplish this, we select the IC locations, the instantaneous motion states of point P of the mobile platform such that the velocity, acceleration and the jerk of point P are instantaneously parallel to each other and tangential to the trajectory at point C thereby eliminating the normal components for acceleration and jerk.

A numerical example of the required motion plan follows.

Step 1: First Order Motion Requirement

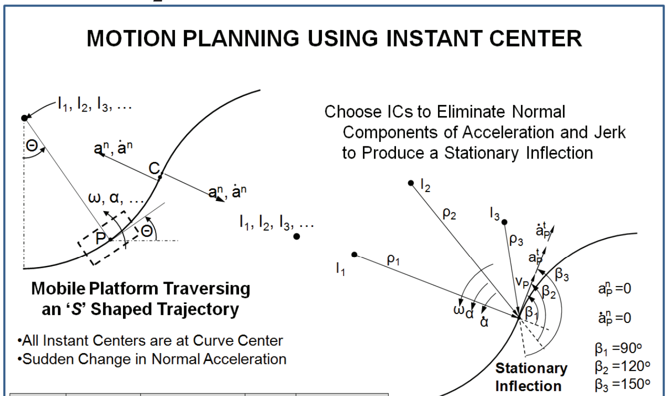
Choose the radius, ρ_1 of point P for the first order IC. Based on ρ_1, compute the magnitude of the angular velocity $\omega = \frac{v_P}{\rho_1}$, where $\beta_1 = \frac{\pi}{2}$ rad. Let the instantaneous linear velocity v_P of point P on the mobile platform be 5 ft/s. Let ρ_1 be 15 ft. Thus the angular velocity, ω, of the platform would be $\omega = \frac{v_P}{\rho_1} = 0.33\frac{\text{rad}}{\text{s}}$.

Step 2: Second Order Motion Requirement

Choose the tangential acceleration a_P^t of point P. Choose the second order orientation angle β_2 which cannot be $\frac{\pi}{2}$ for a nonzero angular velocity, ω. Further, compute the angular acceleration, α, and the radius ρ_2 of P from the second order IC I_2 where $a_P^t = \frac{g}{10}\frac{\text{ft}}{\text{s}^2} = 3.22\frac{\text{ft}}{\text{s}^2}$. Note that a large value of β_2 results in a small value of the angular acceleration and a large value for radius ρ_2. Thus, we choose angle β_2 to be $\frac{2\pi}{3}$ radians (120°). With these numerical values, the instantaneous angular acceleration, α, can be computed as $\alpha = -\frac{\tan\beta_2}{\omega^2} = 0.19\frac{\text{rad}}{\text{s}^2}$ and the radius ρ_2 can be computed as $\rho_2 = -\frac{a_P^t\cos\beta_2}{\omega^2} = 14.48\text{ft}$ (see Fig. 5).

Step 3: Third Order Motion Requirement

Choose the tangential jerk, $\dot{a}_P^t$ of point P required with zero normal jerk. Also, choose the third order orientation angle β_3 which cannot be $\frac{\pi}{2}$ radians for nonzero angular velocity, ω and nonzero angular acceleration, α. In this scenario, we have finite values for both of the terms. Further, compute the radius ρ_3 of P for the third order IC I_3 as well as the angular jerk, $\dot{\alpha}$. Let the linear jerk of point P be 1/20th of the gravitational acceleration constant per second, thus $\dot{a}_P^t = 1.61\frac{ft}{s^3}$. Again, note that a larger value of β_3 results in a smaller value of the angular jerk and a larger value for radius ρ_3. Here, we choose angle β_3 to be $\frac{5\pi}{6}$ radians (150°) as we want a small angular jerk. Thus the instantaneous angular jerk can be computed as $\dot{\alpha} = \omega^3 - 3\omega\alpha \times \tan\beta_3 = 0.15\frac{rad}{s^3}$. Then, the radius ρ_3 of P can be computed using Eq. 2.41 as $\rho_3 = -\frac{\dot{a}_P^t \cos\beta_3}{3\omega\alpha} = 7.24ft$. The numerical values for all the kinematic parameters for the desired motion are given in Fig. 5.



	m_K	Angular Motion	ρ_K	β_K
1st Order	$5\frac{ft}{s}$	$0.33\frac{rad}{s}$	15 ft	$\frac{\pi}{2}$ rad
2nd Order	$3.22\frac{ft}{s^2}$	$0.19\frac{rad}{s^2}$	14.48ft	$\frac{2\pi}{3}$ rad
3rd Order	$1.61\frac{ft}{s^3}$	$0.15\frac{rad}{s^3}$	7.24ft	$\frac{5\pi}{6}$ rad

Avoids Motion Discontinuities In Higher Orders

Figure 5

Based on the observations made during this numerical example, we can provide some guidelines to arrive at useful motion planning values as follows:

1. For a given value of v_P, choosing a large ρ_1 results in a small ω. In other words, $\omega \propto 1/\rho_1$.
2. Choosing a small β_2 reduces the size of ω but increases α. Notice that the magnitude of β_2 can vary between $\frac{\pi}{2}$ and π. For a particular value of β_2, a large ω increases α by the square. When $\beta 2 = \frac{\pi}{2}$, ω is zero.
3. Choosing a small β_2 reduces the size of ρ_2.
4. Choosing a small β_3 (between $\frac{\pi}{2}$ and π) reduces the size of α but increases $\dot{\alpha}$. Notice that the magnitude of β_3 can vary between 0 and π. For a particular value of β_3, a large ω decreases $\dot{\alpha}$ by the cube.

5 .Choosing a small β_3 (between $\frac{\pi}{2}$ and π) reduces the size of ρ_3.

6. Wheel Input Motion Calculations.

This report presents an analytical process to specify with physical meaning the motion of all points E_j j = 1,2,. . .J that are attachment points for active wheel subsystems (preferably centered or offset wheel subsystems). Here, we describe (see Fig. 6) how to obtain the kinematic inputs to an offset wheel subsystem (offset l_j and wheel diameter d_j). Usually, we expect the l_j , d_j to be the same values for all wheel subsystems, although this formulation does not require that simplification.

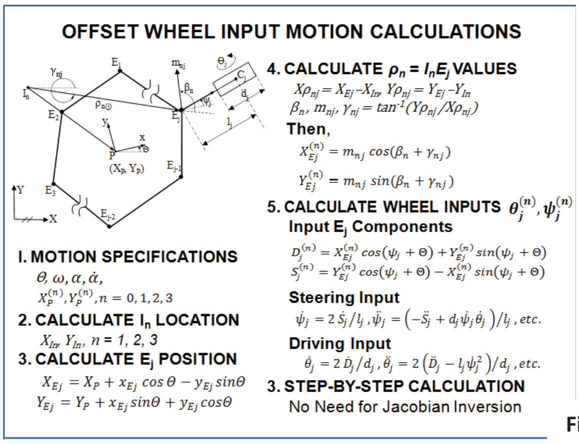


Figure 6

We start with specifications θ, ω, α, ά, X_p^n, Y_p^n, n = 0, 1, 2, 3 up to the n^{th} order. We calculate the location of all needed instant centers (X_{In}, Y_{In}). This allows us to calculate the motion of the wheel attachment points E_j (X_{Ej}, Y_{Ej}) and their higher order properties:

$$X^n_{Ej} = m_{nj} \cos(\beta_n + \gamma_{nj})$$

$$Y^n_{Ej} = m_{nj} \sin(\beta_n + \gamma_{nj})$$

There β_n, m_{nj}, γ_{nj} have been calculated using results presented in Figure 3 and Table 4. This, then, allows us to calculate the two components D_j^n, S_j^n of the motion of E_j parallel to and perpendicular to the wheel offset l_j. Given these values, we can directly calculate the required wheel inputs for steering,.:

$$\dot{\psi}_j = 2\dot{S}_j / l_j$$

$$\ddot{\psi}_j = (-\ddot{S}_j + d_j \dot{\psi}_j \dot{\theta}_j) / l_j$$

Etc.

and for driving :

$$\dot{\theta}_j = 2\dot{D}_j / d_j$$

$$\ddot{\theta}_j = 2(\ddot{D}_j - l_j \dot{\psi}^2) / d_j$$

Etc

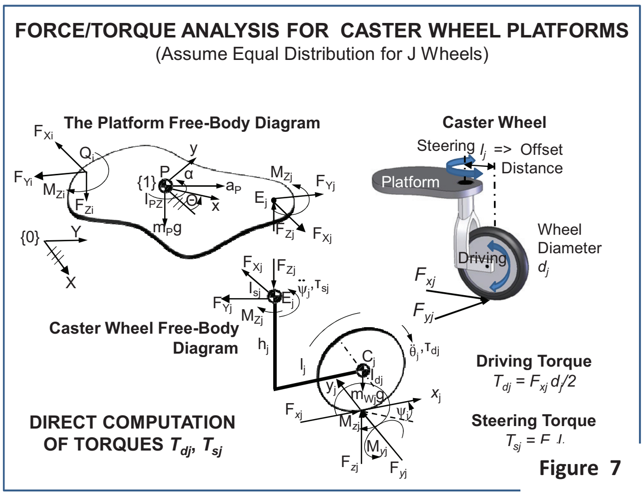


Figure 7

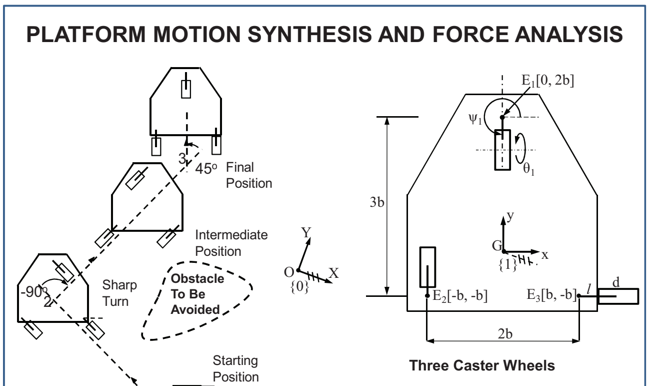
Notice that this step-by-step analytical process results in no mathematical uncertainty which was the result (pseudo inverses) of the Campion formulation. This simple calculation means that we can proceed to the really important problems of mobile platform operation on a simple but sound analytical formulation.

6 Simple Numerical Example.

To demonstrate the steps in the motion synthesis, including required actuator kinematic parameters (θ, $\dot{\theta}$, $\ddot{\theta}$, ψ, $\dot{\psi}$, $\ddot{\psi}$)$_j$ and their required driving torques (T_s, T_w)$_j$ for all j wheel subsystems described in earlier sections, we will illustrate the process in terms of the three caster wheel system shown in Figure 8. The geometric and associated mass properties for the platform are given in the figure.

The platform was required to travel smoothly on a path with the presence of external force/moment.

The goal of the overall motion synthesis was (i) to compute the input velocities ($\dot{\psi}_j$ and $\dot{\theta}_j$) and accelerations ($\ddot{\psi}_j$ and $\ddot{\theta}_j$) required for the platform to complete



l = 1 ft	b = 2 ft	h = 1 ft
m_W = 10 lb	I_{PZ} = 200 lb-ft²	I_d = 10 lb-ft²
d = 1 ft	m_P = 100 lb	I_s = 20 lb-ft²

Figure 8

the motion, and (ii) to compute the input joint torques (τ_{sj} and τ_{dj}) required to sustain the applied and inertia forces/moments acting on the platform during the motion.

The motion programming for the platform is done in using the following steps:

1. Formulate the motion plan for the platform in terms of the linear motion of the Pont Of Interest (POI, in this case it is the centroid of the platform body) and the angular motion of the platform body. Since this motion plan is purely translational, the linear motion of the POI completely describes the motion of the platform. A smooth motion plan such as a trapezoidal shape for the acceleration profile is used for the motion. In this case, the velocity and acceleration of point G is computed for the complete motion plan.
2. Compute the first and second order IC locations for the whole spectrum of the motion plan. In this case, as the motion is purely translational, the first and second order ICs are at infinity during the complete motion. However, in general, this step can provide the user with valuable information as discussed previously.
3. Compute the velocities and accelerations for the three wheel attachment points, E_1, E_2, and E_3, respectively. This should be done using the IC based formulation summarized in Section4.
4. Compute the velocities and accelerations of the control inputs (steering, driving) for each wheel subsystem in terms the steering and driving velocities and accelerations, using the methodology described in Section 6. This completes the kinematic motion synthesis.
5. Next, compute the platform body forces in terms of the applied forces/moments and inertia forces using the free body diagram shown in Fig 7. The applied forces are a result of the payload on the platform as well as the interaction of the platform body with the world.
6. The platform body forces are to be sustained by the set of wheel subsystems. Thus the next step is to distribute these forces among the wheel subsystems. For this numerical example, the forces and moments are distributed evenly among the three wheel subsystems.
7. The next step is to compute the traction force requirements (in longitudinal direction, F_{xj}, and lateral direction F_{yj}) from the ground. These forces must be met by the wheel-ground interaction in order for the wheel to move without slipping/skidding.
8. The last step is to compute the wheel input torques, namely, the driving torque, τ_{sj} and the steering torque τ_{dj} for each wheel subsystem j.

Figure 9 displays the results of the kinematic and dynamic synthesis for the motion trajectory given in Fig. 5.

The kinematic input parameters ($\dot{\theta}$, $\ddot{\theta}$, $\dot{\psi}$, $\ddot{\psi}$) show a unique character (some rapid changes, crossovers, peak-to-peak values, etc.) which illustrates that even for this simple motion plan, the duty cycle on the wheel subsystem can be complex. This is very clearly illustrated by the wheel tractive force curves which are indeed not simple and imply a need for a secure level of traction at the wheel contact surface.

Very similar comments can be made for the resulting actuator torque curves of the wheel subsystem.

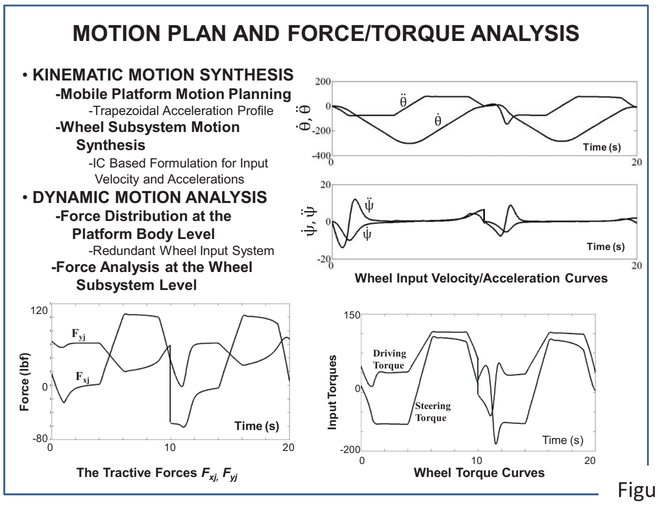


Figure 9

In general, the wheel-ground interaction forces (such as the tractive effort, rolling resistance, lateral (cornering) force, steering resistance (self-aligning force), etc.) are dependent on various external factors such as the vehicle speed v_P, normal force F_{zj}, tire inflation pressure p, tire internal temperature t_i, surface temperature t_s, wetness of the surface characterized by water depth(d), etc. discusses the effect of various operating factors (vehicle speed v_P, normal force F_{zj}, and tire inflation pressure p) on the wheel- friction coefficient in driving and steering directions). These curves further emphasize that these external factors influence the performance of the mobile platforms and should be accounted for while devising the force distribution scheme for successful operation of mobile platforms.

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