

POSITION-DEPENDENT RESPONSE SIMULATION OF MACHINE TOOL USING STATE-SPACE MODELS

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The stability of a machining process is a function of the dynamic response between the spindle and table, which varies within the machine work volume. This paper deals with computationally efficient methodology to evaluate and simulate dynamic performance of the machine tool. A position-dependent virtual model is assembled using finite element model reduced via component mode synthesis and transformed to a state-space multi-input-multi-output system. Combination of these techniques allow time-efficient response simulations with significantly less computational effort than conventionally used full finite element models. The presented approach can be used to create position-dependent dynamic stiffness map within the work volume used to predict and reduce unstable behaviour during operation. Furthermore, these techniques are not reserved for machine tools exclusively and can be used in wider spectrum of technical applications, that require time-efficient response simulations.

KEYWORDS

machine tool, virtual model, component mode synthesis, state-space model, response simulation, position-dependent stability

1 INTRODUCTION

The development of the present machine tool industry requires time-efficient analysis of the structural dynamics and stability of the cutting process. Machine tool's behaviour during operation is directly affected by the dynamic stiffness between the tool and the workpiece, its stability due to chatter constraints depending on various operating parameters such as depth of cut and spindle speed. Furthermore, the dynamic stiffness changes as the tool moves along the path within the machine tools work volume, resulting in position-varying structural vibrations.

The objective during the design process is to maximize dynamic stiffness between the tool and the workpiece while keeping the overall mass of the machine tool low for high-speed positioning and efficient productivity. Up to date, many steps have been made to compensate geometric errors due to the position-dependent static stiffness of the machine during operation [Holub et al. 2016], but unstable behaviour due to the position-dependent dynamic stiffness has not yet been thoroughly described. While static stiffness in one particular point of the machine can be described as a set of numbers in each position within the machine work volume (i.e. stiffness in 3 coordinates), the nature of the dynamic stiffness as a complex function of frequency makes it much harder to determine in a similar way as the static stiffness.

Up to date, problems related to unstable behaviour due to insufficient dynamic stiffness have been mostly solved on existing machines in a way that required some kind of response measurements and compensation [Zaghbani et al. 2009] or simplified analytical models based on spring-damper elements [Siddhpura et al. 2012]. Parameters of the spring-damper system are usually determined by impact hammer measurements with relatively high estimation errors. For complex machine with multiple modes within required frequency range, representation as a 1 degree of freedom (DOF) spring-damper element for each axis is insufficient.

To fulfil demands of modern manufacturing industry, it is necessary to predict and describe dynamic behaviour in the pre-production phase where optimization of the machine takes place, and implement control systems for automatic compensation in later stages of the development, if the dynamic stiffness is still insufficient. However, before it can be achieved, a reliable and time-efficient method to determine dynamic stiffness as a function of position in one or more directions in the global work volume of the machine is required.

2 MODELLING METHODS

The approach presented in this paper is based on complex virtual model of the machine tool [Kšica et al. 2017], which combines multiple modelling techniques in specialized software Ansys and Matlab.

2.1 Reduced FEM Model

Conventional FEM models of large structures usually consist of hundreds of thousands DOF, making them unsuitable for response analyses in time and/or frequency domain. Multiple reduction techniques are available for large-scale FEM models, but only a few are beneficial for dynamic analyses. One of them is component-mode synthesis (CMS), which has these prerequisites:

- system is linear
- response is required in a pre-defined frequency range
- number of reduced DOF is ordinarily lower than number of DOF in full FEM model

CMS is based on an idea that deformation of the structure can be described as linear combination of its modal shapes. This method allows us to disassemble the structure into sub-structures that have common interfaces connected by joints.

Because each sub-system has its own structural matrices, for large-scale system where only some parts are subjected to change, CMS is very beneficial.

Each of these sub-structures is subjected to a modal analysis which considers these interfaces either as free or fixed. Structural matrices (i.e. mass and stiffness) are transformed into more concentrated matrices that contain modal data. As a result, previous structural elements are transformed into so called super elements that no longer contain information in each node of the full FEM mode, but only in user-defined nodes called master nodes. The set of these nodes includes interface nodes, nodes where input and output variables are present, and nodes we want to observe. The requirement that number of reduced DOF (master nodes) is ordinarily lower than number of DEF in full FEM model is necessary for achieving reduction of matrices, because for similar number of DOF, CMS would be counterproductive.

This method is frequently used in dynamic analyses, because of its principle based on modal analysis. An example of CMS method is Craig-Bampton method formulated in 1968, which considers the interface as fixed [Bampton et al. 1968]. CMS is implemented in specialized FEM software such as Ansys. Its application for a simple mechanical structure and comparison of responses of full FEM and reduced FEM models was made [Ksica 2016]. Both full FEM and reduced FEM had within a pre-defined frequency range the same harmonic and impulse response.

2.2 Transfer-function and State-space System

The initial idea is based on the fact that the dynamic compliance (inverse of the dynamic stiffness) is mathematically a transfer function between the input forces and output displacements of the system, as described in equation (1)

$$G(s) = \frac{Y(s)}{U(s)} \quad (1)$$

where G is transfer function matrix representing dynamic compliance, Y is output matrix representing displacement, U is input matrix representing forces, and s is a complex number frequency operator. This transfer function can be easily written as a combination of state-space matrices, as described by equation (2)

$$G(s) = C(sI - A)^{-1}B + D \quad (2)$$

where A , B , C , and D are state-space matrices, and I is unity matrix. For mechanical structure, these state-space matrices can be assembled using modal data (i.e. eigenfrequencies, eigenvectors, and damping). The size of the state-space matrices depends on number of inputs and outputs to the system.

State-space system equations can be written as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (3)$$

where x is the state vector, y is the output vector, u is the input vector, A , B , C , and D are state-space matrices. Consider n number of modes, i number of inputs and o number of outputs. Then A has size of $2n \times 2n$, B has size of $2n \times i$, C has size of $3o \times 2n$ and D has size of $3o \times i$.

System matrix A is defined as

$$A = \begin{bmatrix} 0 & I \\ \Lambda_1 & \Lambda_2 \end{bmatrix} \quad (4)$$

where 0 is null matrix, I is unity matrix. Matrix Λ_1 is diagonal matrix defined as

$$\Lambda_1 = \begin{bmatrix} -\omega_1^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & -\omega_n^2 \end{bmatrix} \quad (5)$$

where ω_j is eigenfrequency of mode j . Matrix Λ_2 is diagonal matrix defined as

$$\Lambda_2 = \begin{bmatrix} -2\xi_1\omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & -2\xi_n\omega_n \end{bmatrix} \quad (6)$$

where ξ_j is effective modal damping of mode j , and ω_j is eigenfrequency of mode j .

Input matrix B is defined as

$$B = \begin{bmatrix} 0 \\ \Lambda_3 \end{bmatrix} \quad (7)$$

where 0 is null matrix. Matrix Λ_3 is defined as

$$\Lambda_3 = \Phi^T F_u \quad (8)$$

where Φ is the matrix of eigenvectors and F_u is a unit force matrix with size $n \times i$, which contains 1 at DOF where input force is active and 0 elsewhere.

Output matrix C is defined as

$$C = \begin{bmatrix} \Lambda_4 & 0 \\ 0 & \Lambda_4 \\ \Lambda_4\Lambda_1 & \Lambda_4\Lambda_2 \end{bmatrix} \quad (9)$$

Matrix Λ_2 is defined as

$$\Lambda_4 = U_u \Phi \quad (10)$$

where U_u is unit displacement matrix with size $o \times n$, which contains 1 at DOF where output is requested and 0 elsewhere.

Matrix D is defined as

$$D = \begin{bmatrix} 0 \\ 0 \\ \Lambda_4\Lambda_3 \end{bmatrix} \quad (11)$$

Presented form of matrix C and D in equations (9) and (11) is valid for system, where not only displacement, but velocity and acceleration is required as an output. If only displacement is required, the last row of matrix C and D is not written.

State-space system is suitable for description of systems with relatively small number of inputs and outputs. In case of machine tools and analysis of their stability during operation, this requirement could be met, because the inputs usually include forces in 3 perpendicular directions applied to the tool, and outputs represent data from sensors (accelerometers), that are fixed at finite and relatively small number of locations.

Because of the complexity of machine tools, they can no longer be represented as single-input-single-output (SISO) system, because there are generally multiple axes of motion. The simplest state-space system for general machine tool would have 3 inputs and 3 outputs, resulting in multi-input-multi-output (MIMO) system.

2.3 Position-dependent Dynamic Stiffness

In case of machine tool, the topology of parts changes as the tool moves along the path. This directly affect static and dynamic stiffness of the machine and its response between the tool and the workpiece. Because the goal of current machine tool development is to identify and minimize the effects of unstable behaviour, such as chatter vibrations, it is crucial to describe the variable dynamic stiffness with enough accuracy. As described above, dynamic stiffness reflects modal behaviour of the machine tool. It is crucial to determine dynamic stiffness for enough positions, as the modes may have multiple extremes within the work volume.

If we used conventional approach of transient analysis using full FEM models, we would be forced to repeatedly run very time-consuming simulations. Furthermore, as the inputs to those simulations are the forces acting upon the system (e.g. cutting forces), which are variables of depth of cut etc., the results would be very limited. Having access to state-space system in each of the crucial positions within the work volume solves this problem, as we can apply any force load as necessary and the response is obtained in seconds, not hours.

Similar approach including position-dependency of the dynamic stiffness was used for an analysis of stability of a vertical mill in [Law et al. 2013], where three vertical positions of headstock were considered. The authors, however, did not use state-space system.

3 VIRTUAL MODEL OF MACHINE TOOL

Previously described methodology is used to create a virtual model of vertical mill, where these parts were included:

- column
- headstock
- spindle
- tool magazine
- drive

The worktable, tool and the tool holding mechanism was not included in the model. Photo of modelled vertical mill is illustrated in Figure 1, respective CAD model of the selected parts is shown in Figure 2.



Figure 1. Photography of modelled vertical mill

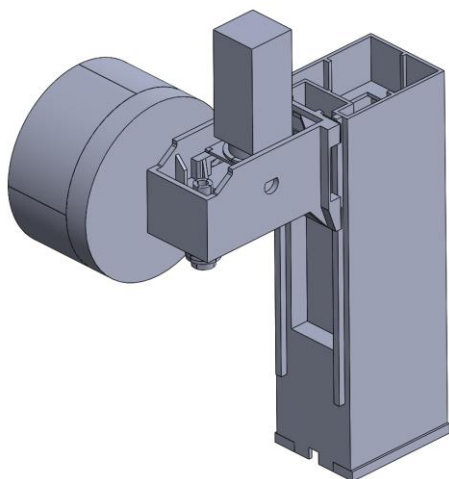


Figure 2. CAD model of vertical mill

3.1 FEM Model

Finite element model of the vertical mill was assembled for the purposes of dynamic simulations. The geometry was simplified, small features (e.g. holes, bevels, chamfers) were removed. Geometry of joints (e.g. ball bearings, ball screws, and linear guides) was not considered and the joints were substituted by mass-less elements. Geometry was cut into separate bodies to allow mapped (swept) meshing. The global model of vertical mill was disassembled into submodels of column, headstock, spindle, drive, and tool magazine, each reduced separately using CMS. The CMS was based on modal analysis with the first 50 modes within range 0-1400 Hz. Super elements were assembled back into a global model, interface nodes were coupled and connected with spring-damper elements. The amount of DOF in full FEM and reduced FEM via CMS is compared in Table 1.

Method	Nodes	Elements	
Full FEM	130,000	270,000	270,000 solid elements
			17 spring-damper elem.
Reduced FEM	8,600	22	5 superelements
			17 spring-damper elem.

Table 1. Number of nodes and elements of full FEM and reduced FEM models

As a separate analysis, effects of the drive belt preload forces and angular velocity of the spindle were evaluated. They did not significantly influence eigenfrequencies and modal shapes ($0 \pm 1\%$). In the following analyses, they were not included as they would introduce complications and different solving algorithm would be necessary.

3.2 Joints

Three types of joints were considered in this model: ball bearings, linear guides, and a ball screw. Because joint stiffness has significant impact on dynamic behaviour of virtual model, it was crucial to determine their parameters with enough accuracy.

Two pairs of high-speed ball bearings with contact angle 25° in QBC configuration were used to connect spindle and headstock. Because ball bearings have two stiffness parameters, axial and radial, they were modelled with planar spring-damper bearing element COMBI214. Because spring-damper elements that are functional only in pressure, are not available in Ansys, axial stiffness of each bearing was determined as a half of axial stiffness of a ball bearing pair. Axial stiffness of each bearing was $k_z = 59.5 \text{ N}/\mu\text{m}$, radial stiffness was $k_r = 119.8 \text{ N}/\mu\text{m}$.

Four linear guides connected headstock and column. Each linear guide has two stiffness parameters in two axes perpendicular to the direction of free motion (x axis and y axis). Therefore, a pair of 1 DOF spring-damper elements COMBIN14 was used for x and y axis. Stiffness in both directions was $k_x = k_y = 77 \text{ N}/\mu\text{m}$.

A ball screw is a relatively complex mechanical system with a lot of parts, which contribute to its overall axial stiffness (z axis). For a fixed-free arrangement, the screw shaft has the lowest rigidity compared to the other parts [Collins 2015], therefore it is possible to exclude the other parts. The shaft of the ball bearing can be modelled as a beam with circular cross section, whose stiffness is a parameter of the distance from the fixed interface. A 1 DOF spring-damper element COMBIN14 was used to model the ball screw, and its variable stiffness was calculated using equation

$$k_z = \frac{E \cdot S}{L} = \frac{E \cdot \pi \cdot D}{4L} \quad (12)$$

where E is Young's modulus of steel, S is cross-section area of the ball-screw, L is the variable distance from the fixed interface, and D is the diameter of the ball screw.

Interface between the column and the magazine, as well as the headstock and the drive, was considered fixed. Position of joints is highlighted in Figure 3.

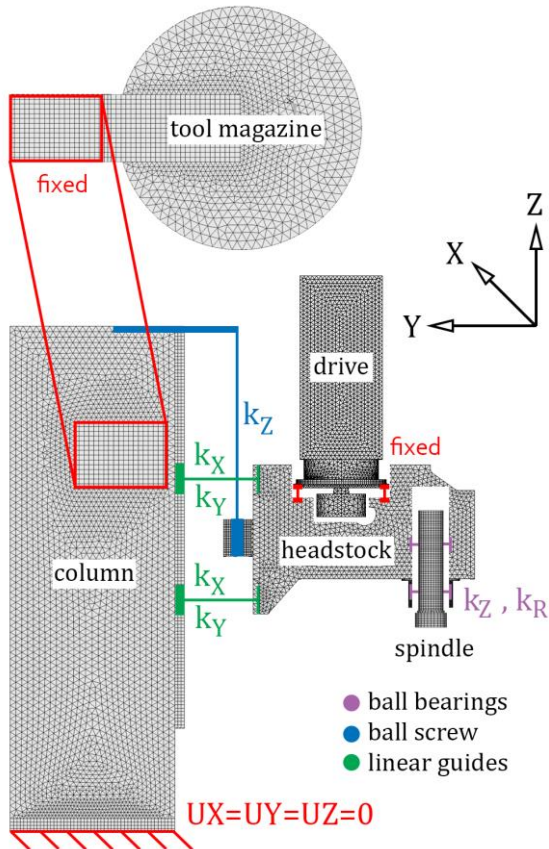


Figure 3. Vertical mill – model of joints

3.3 Damping

Evaluation of damping in complicated machinery has always been a problem. To preserve simplicity, at this stage of research, damping was not included. However, as the structure of state-space matrices implies (equation (4) – (11)), it is possible to easily include modal damping. The damping coefficients can be either based on similar technical problems, or the existing machine tool can be subjected to a response measurement and for each eigenfrequency, modal damping can be determined.

4 STATE-SPACE SYSTEM

The main idea behind obtaining state-space system of the vertical mill without a tool attached is based on the assumption that we can connect multiple state-space systems (or transfer functions) in a chain. In case of machine tools, we want to observe response of tool – workpiece pair, but we are usually unable to measure any mechanical parameters precisely on the tool during operation. If we were to know transfer function between a tool and a sensor attached somewhere else on the machine, we would easily add that to a response of tool and workpiece itself. This is illustrated in Figure 4.

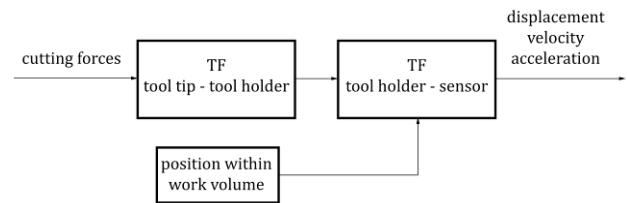


Figure 4. Chain of transfer functions

The problem is, however, more complicated due to the dependence on the current position within work volume, resulting in a series of state-space systems, one for each position respectively.

Therefore, an algorithm that selects the state-space system for the respective position is necessary. In order to assemble state-space matrices A, B, C, and D, reduced FEM model is subjected to a series of modal analysis to extract eigenfrequencies, eigenvectors and damping. Pre-defined range of frequencies was 0÷900 Hz, in which 30 modes were computed. As inputs, acting forces on tool holder in a global coordinate system were chosen, and as outputs, displacements in a global coordinate system of a user-defined sensor location were chosen (Figure 5). A set of positions within range of 0÷600 mm with step size of 5 mm was used.

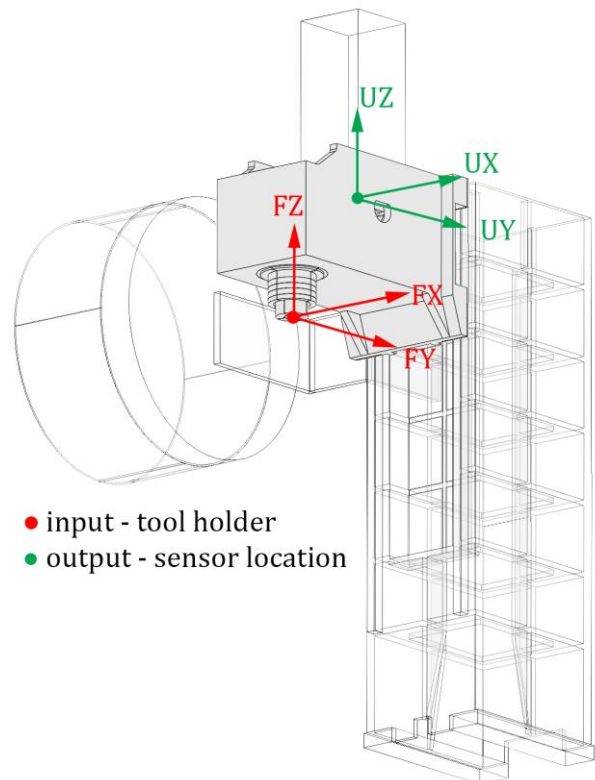


Figure 5. Defined inputs and outputs

Each extracted state-space system is loaded into Matlab as a part of a structure, that among the matrices A, B, C, and D contains position, eigenvalues, eigenvectors and modal damping, as illustrated in Figure 6. This structure serves as a database of state-space systems for each position within work volume, currently only for vertical axis z.

Fields	z_position	A	B	C	D	eigenvalues	eigenvectors	damping
1		60x60 double	60x2 double	6x60 double	6x3 double	30x1 double	30x3 double	30x1 double
2		-0.0050 60x60 double	60x2 double	6x60 double	6x3 double	30x1 double	30x3 double	30x1 double
3		-0.0100 60x60 double	60x2 double	6x60 double	6x3 double	30x1 double	30x3 double	30x1 double
4		-0.0150 60x60 double	60x2 double	6x60 double	6x3 double	30x1 double	30x3 double	30x1 double
5		-0.0200 60x60 double	60x2 double	6x60 double	6x3 double	30x1 double	30x3 double	30x1 double

Figure 6. Matlab structure with position-dependent state-space matrices

A selection algorithm picks a respective state-space system for an input position within the work volume. These state-space matrices are loaded into a simple Simulink model (Figure 7), that simulates response for the input forces.

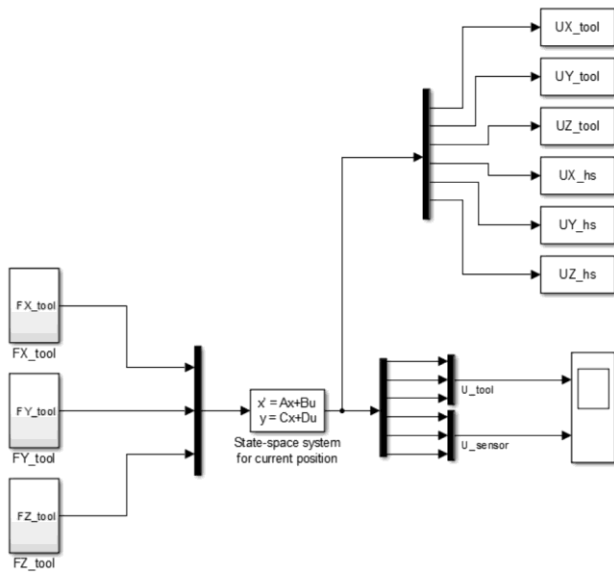


Figure 7. State-space model of tool holder – sensor in Simulink

Because the state-space systems are extracted for discrete positions, the transition between them is not smooth. The selection algorithm is based on a ceiling function and the transition is stepped. However, the position steps were small enough to make the transitions as smooth as possible. In the future, a state-space system interpolation method will be included [Caigny et al. 2011].

5 RESULTS

Presented algorithms were used to simulate the response of the machine tool as an example of their functionality. To illustrate the necessity to determine response as a function of position, a graph containing eigenfrequencies of the machine based on vertical position of the headstock was made (Figure 8).

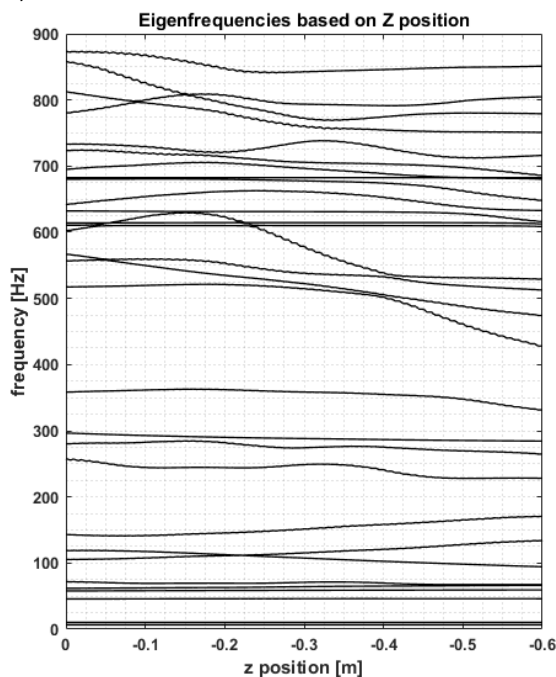


Figure 8. Eigenfrequencies for variable position of the headstock

For several modes, minimum frequency, maximum frequency, and percentual difference is compared in Table 2.

Mode number	Minimum	Maximum	Difference
	[Hz]	[Hz]	[%]
5	61.7	66.3	7.5
6	67.4	72.0	6.8
7	105.4	134.4	27.5
8	94.4	119.2	26.3
9	140.9	170.9	21.3
16	428.2	566.6	32.3
17	512.8	629.6	22.8

Table 2. Comparison of eigenfrequencies for variable position of the headstock

As can be seen, for some modes, the frequency shift due to the variable position can be over 30 %. Furthermore, some modes exhibit extremes in between the topmost (0 m) and the bottommost (-0.6 m) position. It can be seen that describing dynamic behaviour only in the topmost and bottommost position of the headstock can introduce significant errors and linear interpolation between the end positions is out of the question.

The response simulation of the extracted state-space systems was done in Matlab/Simulink environment. This choice was made because this software allows easy work with state-space systems, their transformation into transfer functions and it can be used as a base for future simulations, where response of the tool and workpiece is included. Also, since some of the interfaces can exhibit behaviour that is hard to describe in Ansys (e.g. friction, backlash etc.), additional joint functions can be added between the subsystems in Simulink.

As mentioned above, position-dependent response of the system with unitary force impulse as inputs was made. Impulse peak was 1 N and impulse length was 1 ms, unitary force was applied simultaneously in all three global directions. Firstly, a time-domain response (FRF) using Fast Fourier transform (FFT) was made (Figure 10). FFT data were subjected to an average filter with kernel size of 5 samples for smoothening.

It is important to mention that each response simulation took only seconds to finish, compared to the alternative which might include full transient or harmonic analysis in Ansys, and that would take hours instead.

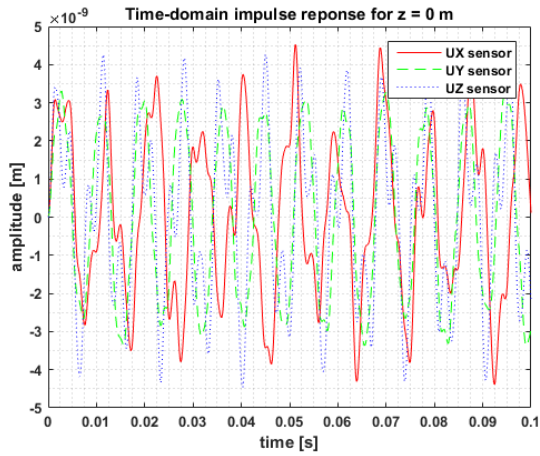


Figure 9. Time-domain impulse response for the topmost position

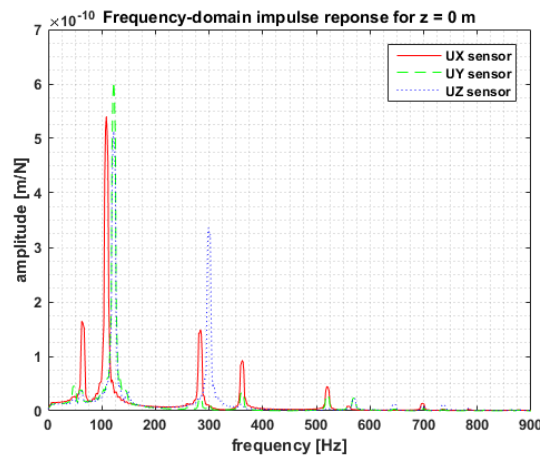


Figure 10. Frequency-domain impulse response for the topmost position

These simulations were done for each position step on the vertical axis. The position vector was added as a third axis to a 3D plot of the final FRF results. In Figure 11 - Figure 13, shift in amplitudes can be compared. In Figure 14 - Figure 16, shift in frequency can be compared. These figures are similar to the Figure 8, but show only those modes that were excited by the input forces.

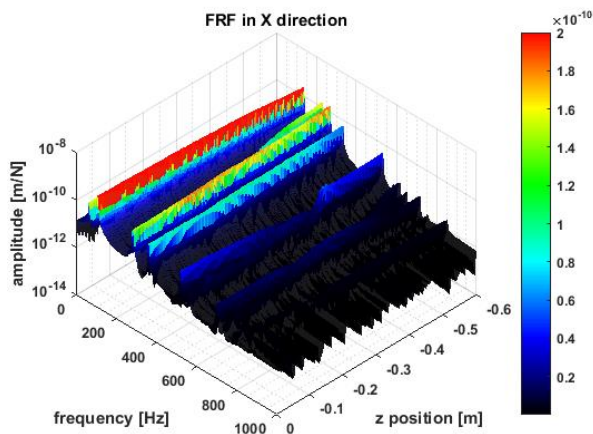


Figure 11. FRF in X direction, amplitude view

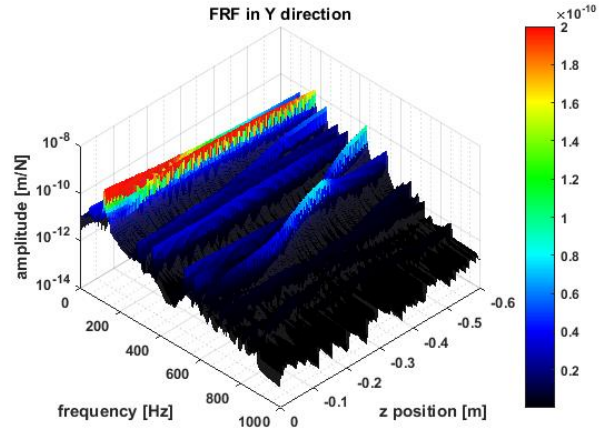


Figure 12. FRF in Y direction, amplitude view

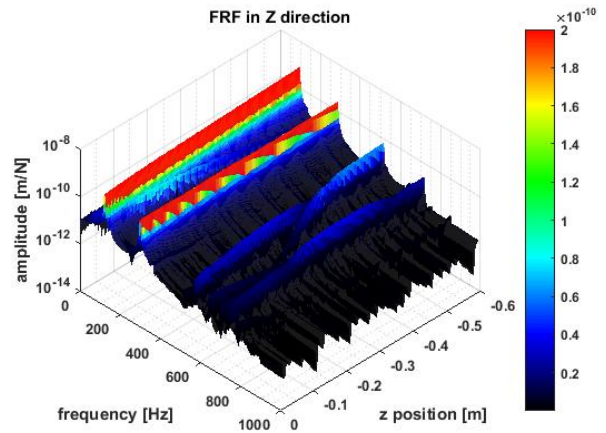


Figure 13. FRF in Z direction, amplitude view

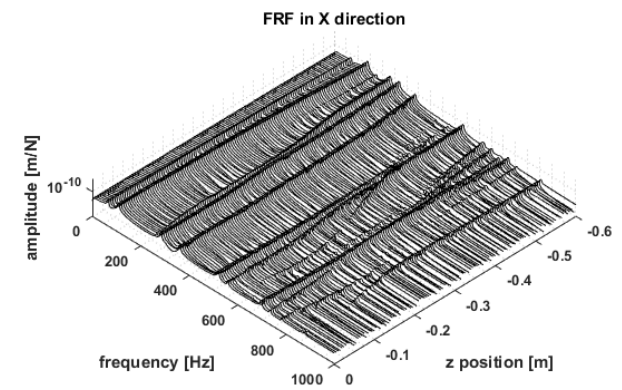


Figure 14. FRF in X direction, frequency view

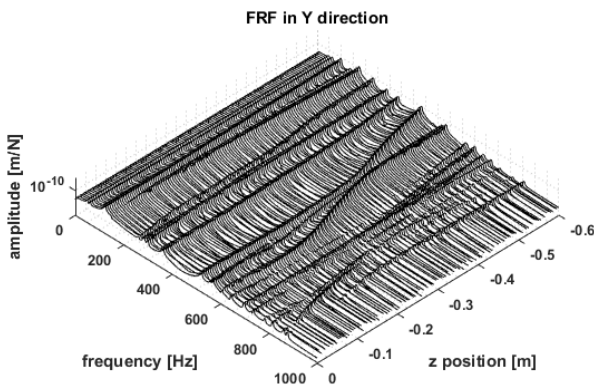


Figure 15. FRF in Y direction, frequency view

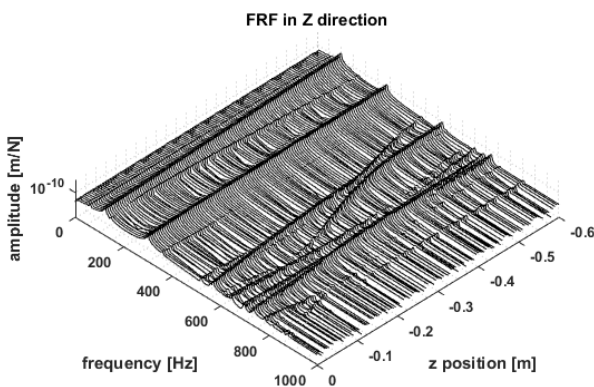


Figure 16. FRF in Z direction, frequency view

As can be seen, some modes have amplitude as well as frequency extremes between the topmost and the bottommost positions, and such phenomenon is hard to predict when simulations are made only for the end positions.

6 DISCUSSION

Proposed method based on transformation of full FEM or reduced FEM model into a state-space system proved to be viable for response simulations, where time is crucial. This paper focused mainly on application of this approach for a real problem, however, some questions might emerge.

Firstly, there might be a question of linearity. Modal analysis can be done only for linear systems, meaning systems that do not introduce nonlinear behaviour due to material, joints, large deflections etc. For an application that involves machine tool, this is might not seem as a significant limitation. Noticeable nonlinear behaviour may be introduced in joints, mainly as friction and backlash, but these effects can be implemented separately to the Simulink model. Furthermore, state-space and transfer-function based models only work for linear time-invariant systems (LTI)[Brezina et al. 2012].

Secondly, there is a question of experimental validation of this approach. These methods were previously discussed in a diploma thesis [Kšica 2016], where some techniques that might be used for experimental validation were described. Experimental validation is based on measurement of some key parameters that might be directly or indirectly compared with the validated virtual model. In case of dynamics, these parameters might be, among other things, eigenfrequencies, amplitudes of vibrations, and damping. Time-domain response comparison is not suitable for validation, as it is subjected to noise, phase shift, and overall it is difficult to compare

objectively. However, time-domain data can be used for dynamic system identification to create a mathematical model (e.g. transfer function, state-space model) of the system that could be used for response comparison for different, more suitable inputs. Dynamic system identification is usually based on the least-squares method, where parameters of the chosen mathematical model are tuned to match the target measured response [Ljung 2007]. For SISO systems, the identification process is rather straightforward. However, for MIMO systems it becomes more complicated because we are tuning parameters for $n \times m$ different combinations, where n is the number of inputs and m it the number of outputs. Frequency-domain response might be used to adjust eigenfrequencies and more importantly damping, which might be calculated for each mode separately.

Thirdly, a question related to the implementation of tool tip – tool holder response and cutting forces, that would reflect real system. A tool in its simplest form can be substituted by a beam, for which a response is relatively easy to compute. This beam can be substituted by a more detailed geometry, but it would ultimately be based around the same idea that this paper discusses. Implementing cutting forces to simulate machining process would only mean to substitute unity force impulses with more complicated functions [Özs et al. 2015][Putz et al. 2016].

7 CONCLUSIONS

This paper introduced methods for virtual modelling of machine tool based on transformation of reduced FEM models into state-space domain. Because the stability of the cutting process is directly affected by the dynamic stiffness of the machine tool, it is important to describe it with enough accuracy. The focus was position-dependent dynamic stiffness that significantly affects frequencies and amplitudes of excited vibrations, and its use for time-efficient response simulation for specific working conditions of the machine.

Mechanical structure of a vertical mill can be disassembled into separate substructures (e.g. column, headstock, spindle etc.) and modelled as single entities. In the Simulink model, they are connected using direct joints or joints functions in much simpler manner than in FEM software.

Firstly, a reduced FEM model of each part of a vertical mill using CMS method was created in Ansys. This significantly reduced number of nodes and elements of the FEM model and drastically decreased time required for subsequent analyses.

Secondly, this reduced model was transformed into state-space system using modal analysis. The state-space system had three defined inputs, forces in each of the global axes respectively. The outputs were displacement of the structure in one chosen location, which is an assumed location of a sensor. This location can be easily changed to match sensor location on an existing machine, as the model is fully parametric. These state-space systems were calculated for each position step on a vertical axis of the headstock and loaded into Matlab, where a complex structure with state-space matrices, eigenfrequencies, eigenvectors and damping was assembled. A selection algorithm was used to find state-space system for the specific position within database and unitary force response simulation was made using model in Simulink.

Presented method proved to be more than viable for response simulations where time is crucial. The goal of this paper is to

propose an algorithm to create a state-space system (transfer function) map within the work volume, that might be used for adjustments of speed, depth of cut and an input to regulation algorithm to minimize effects of unstable vibrations (i.e. chatter) during operation. The machine would calculate its path, segmented this path into steps, for each segment find respective state-space system within its database for the current tool, set required speed and depth of cut, calculate cutting forces and response of the system and forward it to the regulation algorithm, that would adjust necessary parameters to maximize precision and speed.

Presented technique is not limited to use for machine tools exclusively, it can be implemented in various areas including energy harvesting and aeronautics. It is a subject of an ongoing research, which aims to provide a general algorithm for time-efficient response simulations of complex structures, mainly in the manufacturing industry.

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