ACCELERATED SUBSPACE ITERATION METHOD FOR SIMULATING VARYING EIGENPAIRS OF THIN-WALLED WORKPIECES

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This paper presents an innovative accelerated subspace iteration method tailored for simulating varying eigenpairs of thin-walled workpieces. The proposed approach significantly boosts computational efficiency by selecting optimal start iteration vectors for a given cutter location. The optimal start iteration vectors are generated based on the previously calculated eigenvectors from the preceding cutter location. A numerical example illustrates the efficiency enhancement achieved by the proposed method, showcasing a notable reduction in computation time. Furthermore, to evaluate the numerical accuracy of the accelerated subspace iteration method, comprehensive comparisons are conducted with results obtained from Abaqus simulations. The outcomes affirm the numerical accuracy of the proposed approach.

KEYWORDS

thin-walled workpiece, eigenvalues, eigenvectors, eigenpairs, subspace iteration method, start iteration vectors, finite element simulation

1 INTRODUCTION

CAM planning for thin-walled workpieces poses substantial challenges [Do 2018] [Ratchev 2005] [Sun 2023]. A critical issue in this domain is the significant vibration encountered during the machining process [Dang 2021] [Karimi 2022] [Karimi 2024] [Liu 2022] [Tehranizadeh 2022]. To avoid the large vibration, optimal selection of process parameters is necessary. However, the conventional approach to parameter refinement entails numerous machining tests and iterative adjustments, resulting in substantial material wastage and time consumption.

A promising approach involves utilizing numerical methods to predict the dynamic characteristics of thin-walled workpieces, which can then guide the selection of optimal process parameters [Karimi 2022] [Karimi 2024] [Tuysuz 2017] [Tuysuz 2018]. The essence to predict the dynamic characteristics of a mechanical structure is the solution of the first p eigenpairs of the generalized eigenproblem [Bathe 2014]:

$$K\Phi = M\Phi\Lambda \tag{1}$$

with:

K: $n \times n$ system stiffness matrix

n: number of degrees of the freedom (DOF)

Φ: $[\phi_1, \dots, \phi_p]$, first p eigenvectors

M: $n \times n$ system mass matrix

Λ: $diag(\lambda_i), i = 1, \dots, p$, first p eigenvalues.

Bathe's subspace iteration method is widely used to solve Eq. 1 because of its high efficiency. This method consists of the following main steps [Bathe 2013] [Bathe 2014]:

1. Set the number of required eigenpairs p.
2. Set the number of the iteration vectors q equal to $max\{2p, p+8\}$.
3. Generate q start iteration vectors X_1:
$$X_1 = [x_{1,1}, x_{1,2}, \dots, x_{1,q}] \tag{2}$$
4. Solve the following equations for $k = 1,2, \dots$:
$$K\bar{X}_{k+1} = MX_k \tag{3}$$
$$K_{k+1} = \bar{X}_{k+1}^T K\bar{X}_{k+1} \tag{4}$$
$$M_{k+1} = \bar{X}_{k+1}^T M\bar{X}_{k+1} \tag{5}$$
$$K_{k+1}Q_{k+1} = M_{k+1}Q_{k+1}\Lambda_{k+1} \tag{6}$$
$$X_{k+1} = \bar{X}_{k+1}Q_{k+1} \tag{7}$$

with:

X_k ($n \times q$ matrix): approximation of the eigenvectors in iteration $k-1$

$\bar{X}_{k+1}$ ($n \times q$ matrix): intermediate matrix to calculate K_{k+1} and M_{k+1}

$\bar{X}_{k+1}^T$ ($q \times n$ $matrix$): transpose of $\bar{X}_{k+1}$

K_{k+1} ($q \times q$ matrix): reduced stiffness matrix

M_{k+1} ($q \times q$ matrix): reduced mass matrix

Q_{k+1} ($q \times q$ matrix): eigenvectors of the reduced eigenproblem Eq. 6

Λ_{k+1} ($q \times q$ diagonal matrix): eigenvalues of the reduced eigenproblem Eq. 6

X_{k+1} ($n \times q$ matrix): approximation of the eigenvectors in iteration k

From step 1 to step 3, using a larger number of iteration vectors q compared to the number of required eigenpairs p facilitates accelerated convergence in Step 4, as noted by Bathe [Bathe 2013]. In step 4, the original eigenproblem Eq. 1 of order n is transformed into the eigenproblem Eq. 6 of order q. For most engineering problems, the number of required eigenpairs is considerably smaller than the system size n. Hence, the eigenproblem Eq. 6 can be efficiently solved by using the Jacobi solution method [Bathe 2014]. The required eigenpairs are iteratively approximated from Eq. 3 to Eq. 7.

It's important to note that one of the most time-consuming computations in the subspace iteration method is the solution of $\bar{X}_{k+1}$ in Eq. 3. This computation requires the Cholesky decomposition of K [Scott 2023]:

$$K = LL^T \tag{8}$$

with:

L: $n \times n$ lower Cholesky factor

L^T: $n \times n$ upper Cholesky factor

Furthermore, the efficiency of the iteration process between Eq. 3 and Eq. 7 is tied to its convergence rate, having a direct impact on computation time. The inherent drawback of the subspace iteration method lies in the possibility for slow convergence, leading to high computation time. However, this drawback is counterbalanced by the method's ability to be highly efficient when convergence occurs rapidly [Bathe 2013] [Bathe 2014].

The special challenge in predicting the dynamic characteristics of thin-walled workpieces using the subspace iteration method arises from the variation of the workpieces' eigenpairs caused by material removal during the machining process. To consider the variation, the eigenproblem Eq. 1 is required to be solved for multiple intermediate states of the workpiece. This requires generation of system stiffness matrix K and system mass matrix M for these intermediate states. The system stiffness matrix of a workpiece during machining process can be generated by using the following procedures [Brecher 2023]:

1. Mesh the initial geometry of the thin-walled workpiece.
2. Store the elemental stiffness matrices and the system stiffness matrix $K_{sys,1}$ of the initial workpiece.

3. Detect the deleted finite elements after each cutting step.
4. Update the system stiffness matrix for $q = 2,3, \ldots$:

$$K_{sys,q} = K_{sys,q-1} - \sum_{i} K_i \quad (9)$$

with:

$K_{sys,q}$: system stiffness matrix of the mesh q

$K_{sys,q-1}$: system stiffness matrix of the mesh $q-1$

K_i: elemental stiffness matrix of the removed element i

The outlined procedures can also be employed to generate the system mass matrix. However, considering the varying system stiffness matrix and system mass matrix, solving eigenproblems for multiple intermediate states of the workpiece using the subspace iteration method can require a large computation time. This limits the application of the numerical method to support the process parameter optimization in the industry.

This paper primarily seeks to enhance the convergence rate of the subspace iteration method in calculating varying eigenpairs of thin-walled workpieces. Following this introductory section, section 2 outlines a strategy for augmenting the convergence rate by generating optimal start iteration vectors for the subspace iteration method. Subsequently, section 3 systematically evaluates both the efficiency and the accuracy of the accelerated subspace iteration method. The final section summarizes the key findings and outlines future research directions.

2 ACCELERATION OF THE SUBSPACE ITERATION METHOD BY USING OPTIMAL START ITERATION VECTORS

This section presents a method to increase the efficiency of the subspace iteration method in solving the eigenproblems for thin-walled workpiece. One important property of the subspace iteration method is:

- If the start iteration vectors are the required eigenvectors or linear combinations of the required eigenvectors, the subspace iteration method converges in one step [Bathe 2014].

This property leads to the following hypothesis:

- Hypothesis 1: The convergence rate of the subspace iteration method can be increased by minimizing the disparity between the start iteration vectors and the real eigenvectors.

It is import to note that the machining process of a workpiece is a continuous process. This leads to the second hypothesis:

- Hypothesis 2: The material removal during a machining process leads to a continuous variation of the workpiece's mass and stiffness properties. Consequently, the difference between two successive intermediate states of the workpiece, with regard to their mass and stiffness properties, is minimal. As a result, the corresponding eigenvectors exhibit only minor disparities.

Assuming the validities of the hypotheses, the convergence rate of the subspace iteration method to simulate an intermediate state q of a thin-walled workpiece can be increased by using the following process:

1. Generate start iteration vectors for intermediate state q by selecting entries from eigenvectors Φ_{q-1} of intermediate state $q-1$. The chosen entries correspond to the common DoF of meshes q and $q-1$.
2. Solve the eigenproblem Eq. 1 for intermediate state q to get eigenvectors Φ_q and eigenvalues Λ_q using the subspace iteration method. The optimal start iteration vectors selected in the previous step accelerate the convergence of the subspace iteration method.

This process is illustrated in Fig. 1, where each node has a single DoF in vertical direction. Mesh $q-1$ comprises 10 nodes and 10 corresponding DoF. The eigenvectors $\Phi_{q-1} \in \mathbb{R}^{10\times 2}$ are known from a prior simulation. Mesh q is derived by removing node N1 from the mesh $q-1$. The entries in Φ_{q-1} corresponding to the remaining nodes are selected to establish start iteration vectors $\Phi_q^1 \in \mathbb{R}^{9\times 2}$ for mesh q. The start iteration vectors Φ_q^1 are used to calculate the eigenvectors $\Phi_q \in \mathbb{R}^{9\times 2}$ for mesh q. Given the small difference between the meshes q and $q-1$, Φ_q^1 is expected to be a reliable approximation of Φ_q, facilitating a high convergence rate of the subspace iteration method used for the mesh q.

It's necessary to acknowledge that the method for generating optimal start iteration vectors for mesh q utilizes the calculated eigenvectors of the previous mesh $q-1$ as input. Consequently, this method is not applicable to the first mesh. Addressing this special case for the first mesh and the normal case for the other meshes, the full process to calculate eigenpairs for all intermediate meshes of a thin-walled workpiece involves the following steps:

- Generate system stiffness matrices and system mass matrices for all workpiece's intermediate states by using the matrix updating procedures explained in the introduction section [Brecher 2023].
- Calculate the eigenpairs including eigenvectors Φ_1 and eigenvalues Λ_1 for $q = 1$ (first mesh) with start iteration vectors established by using the standard method of Bathe [Bathe 2014].
- For $q = 2, \ldots, Q-1, Q$, where Q is the total number of the intermediate meshes:
  - Generate the start iteration vectors Φ_q^1 based on Φ_{q-1}^1.
  - Calculate eigenvectors Φ_q and eigenvalues Λ_q using the subspace iteration method with the generated start iteration vectors Φ_q^1.

The presented section introduces a concise approach to generate the optimal start iterating vectors for the subspace iteration method in simulating thin-walled workpieces. The subsequent section evaluates the efficiency and accuracy of the subspace iteration method when employing the well-established start iteration vectors, demonstrated through a numerical example.

3 NUMERICAL TEST

In this section, the accelerated subspace iteration method introduced in the previous section is evaluated through a numerical example involving the flank milling of a thin-walled workpiece, as illustrated in Fig. 2. The raw workpiece is a $50\ mm \times 6\ mm \times 40\ mm$ thin wall. 7 Z-layers with a total thickness of $1.9\ mm$ are removed from the workpiece during machining process. All Z-layers have the same height $40\ mm$. Each Z-layer is removed by 267 cuts. The offset in Y-direction between two successive cuts is $0.15\ mm$.

The material removal during a real milling process occurs continuously, resulting in an infinite number of intermediate states for the workpiece. However, it is impractical to simulate all of these intermediate states. To ensure accurate simulation results without incurring excessive computation time, it's crucial to achieve a balance in selecting the number of simulated intermediate states. While an extensive simulation with numerous intermediate states enhances precision, it also escalates computational demands. Conversely, opting for too few intermediate states reduces accuracy. In this numerical

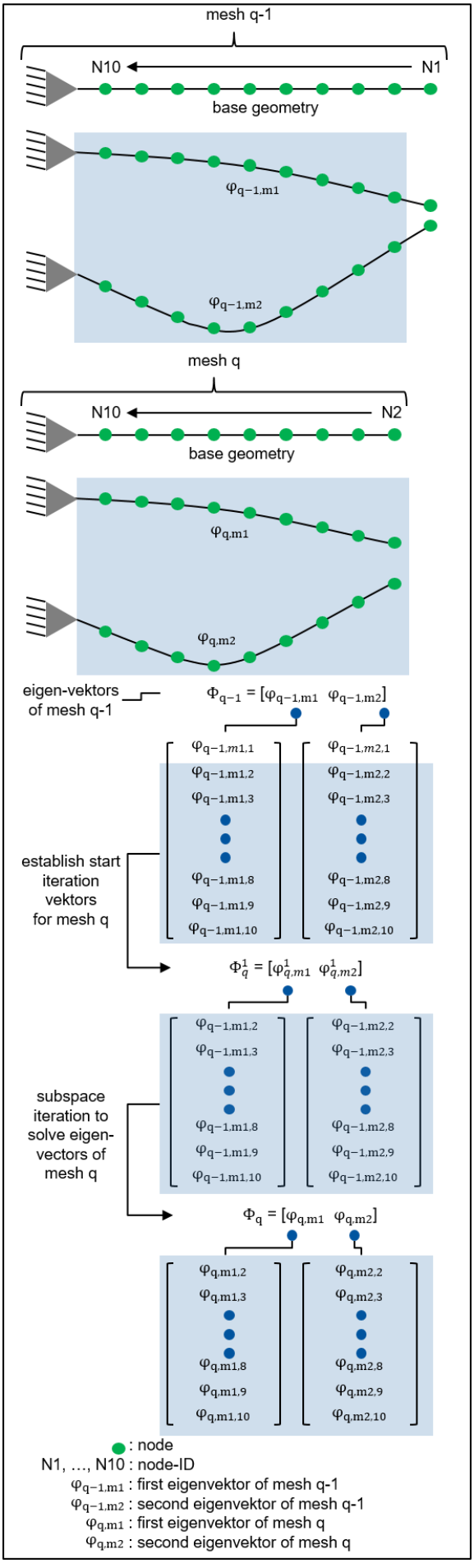

Figure 1: Generation of an optimal start iteration vector

example, 19 intermediate states corresponding to 19 equidistantly distributed cutter locations (CL) are simulated for each Z-layer. Hence, the whole machining process of the workpiece requires $19 \times 7 = 133$ simulations. In the first simulation, the eigenproblem Eq. 1 has 75,744 DoF. The number of DoF decreases to 53,904 in the last simulation due to material removal. The first 10 eigenvalues and the corresponding 10 eigenvectors are calculated in each simulation.

In Fig. 3, the time usage for each cutter location (CL) is detailed. As explained in the introduction section, the two primary time-consuming components in the subspace iteration method are

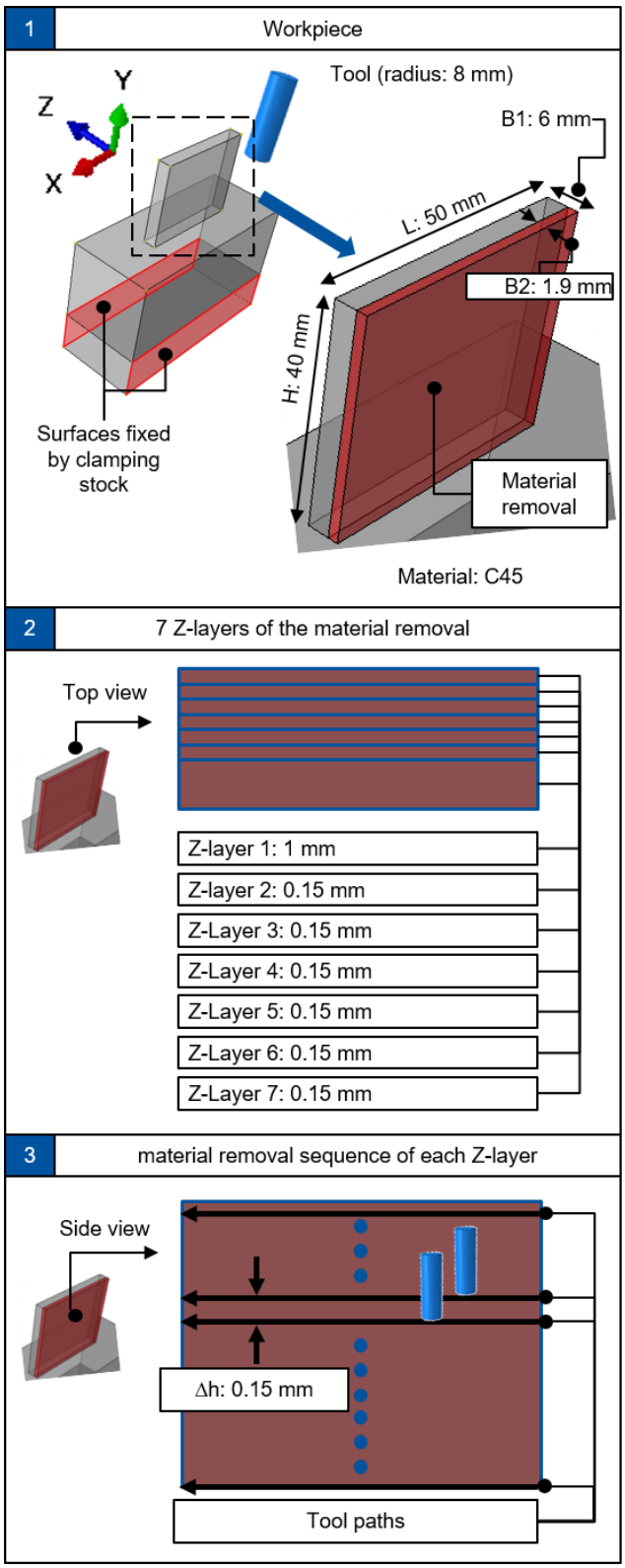

Figure 2: Numerical example

the iterative approximation of eigenpairs (from Eq. 3 to Eq. 7) and the Cholesky decomposition (Eq. 8). The corresponding time usages are denoted as iteration time and Cholesky decomposition time. The total time of the subspace iteration method is the sum of the iteration time and the Cholesky decomposition time. As depicted in the lower part of Fig. 3, utilizing optimal start iteration vectors reduces the number of iterations from more than 15 to less than 5. This results in a significant reduction of iteration time, as shown in the upper part of Fig. 3. However, the Cholesky decomposition time is solely dependent on the system stiffness matrix. Hence, the curves depicting the Cholesky decomposition time with and without optimal start iteration vectors are overlapped.

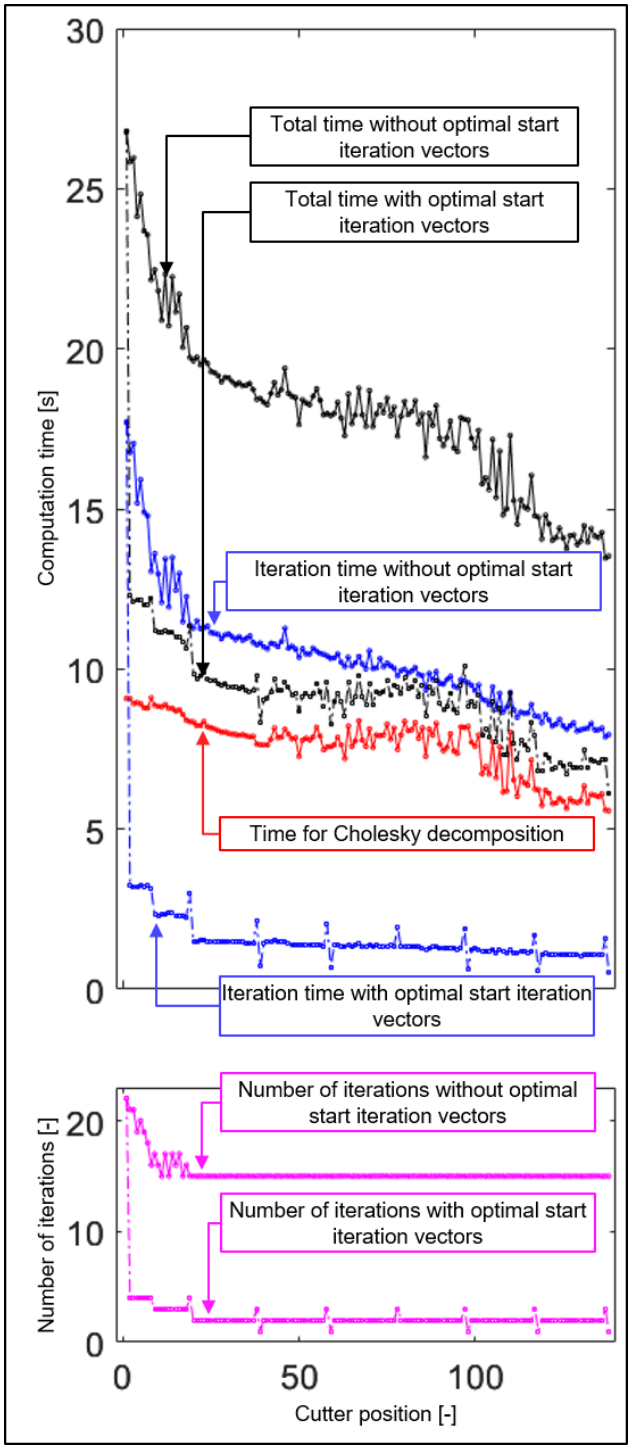

Figure 3: Time to compute the eigenpairs

Consequently, the total time reduction (approximately 50 %) for the simulation is realized by the reduction of the iteration time.

The accuracy of the subspace iteration method, accelerated by using optimal start iteration vectors, is evaluated by comparing it to the commercial software Abaqus. The first two natural frequencies of all 133 intermediate states are calculated with Abaqus and presented in the upper part of Fig. 4. The results from Abaqus serve as reference. Upon comparing the results of the accelerated subspace iteration method to those of Abaqus, a small difference is observed within a narrow tolerance range from $-0.1\ Hz$ to $0.1\ Hz$ in the lower part of Fig. 4. Therefore, the numerical accuracy of the accelerated subspace iteration method is confirmed.

In this section, the efficiency of the accelerated subspace iteration method is demonstrated by a significant total time reduction of about 50 %. Additionally, the numerical accuracy of this accelerated method is confirmed through comparison with the commercial software Abaqus.

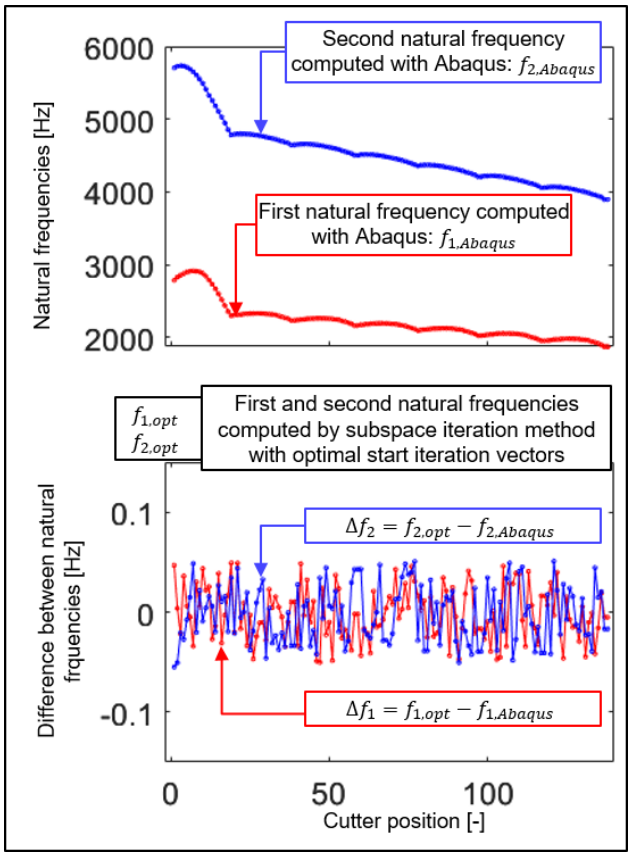

Figure 4: Accuracy of the accelerated subspace iteration method

4 CONCLUSIONS

This paper presents a concise and efficient approach for simulating the varying eigenpairs of thin-walled workpieces using an accelerated subspace iteration method. The method's efficiency is notably enhanced by establishing optimal start iteration vectors based on the simulated eigenvectors from preceding cutter locations. Comparative analysis demonstrates its outstanding efficiency compared to the subspace iteration method without optimal start iteration vectors, while the numerical accuracy level of the commercial software Abaqus is achieved. The combination of high efficiency and good accuracy of this novel approach makes it well-suited to predict the varying dynamic characteristics of thin-walled workpieces, thereby facilitating the selection of optimal process parameters.

In future work, the method will be applied to real workpieces during CAM planning to validate its efficiency and accuracy under practical conditions.

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