METHODOLOGY OF ROTATION OF GENERAL SURFACES

RADEK LEHNER[1], JAN VALDMAN[2], HELENA ZIDKOVA[3]

[1]UNIVERSITY OF WEST BOHEMIA, Faculty of Mechanical Engineering, Pilsen, Czech Republic
[1]SMT a.s. COMPANY, Pilsen, Czech Republic
[2]UNIVERSITY OF SOUTH BOHEMIA, Faculty of Science, Ceske Budejovice, Czech Republic
[2]UTIA AV CR, v.v.i., Prague, Czech Republic
[3]UNIVERSITY OF WEST BOHEMIA, Faculty of Mechanical Engineering, Pilsen, Czech Republic

DOI: 10.17973/MMSJ.2024_12_2024037

radek.lehner@cz-smt.cz

When parts are machined, especially on heavy machine tools, an accurate description of the rotations around the geometric axes is required. The work adds the concept of rotation around the machine and the workpiece axes. Both types of rotation in mechanics are related to internal and external rotations, which we describe in detail. We show how both types convert to each other and how conversion formulas are derived. The resulting conversions are formalized as functions in the C and MATLAB programming languages. The result of this mathematical description will be transferred to a new transformation cycle that can be used on machines as a standard.

KEYWORDS

manufacturing, geometric and machine axes, intrinsic and extrinsic rotations, Cycle 800, workpiece probe, rotary table, milling head

1 INTRODUCTION

A discussion of the use of polynomial transformations by five-axis machining can be found in [Ohnistova 2018]. It deals with a description of spline interpolation on surfaces. Information on the use of new devices for rapid measurement of geometric errors of machine tools is described in [Kuprin 2022]. The measurement of these errors and information on how to use a volumetric compensation on machine tools is provided in [Holub 2015]. The basis rotations of surfaces are different from the focus of these publications. No source was found to describe transformations (translation and rotation) on machine tools using the milling heads, rotary tables, positioning equipment, and their combinations. It is practically used in cycles of the SINUMERIC operation system (Traori or Cycle 800) [Siemens manual 2017] without more detailed information. In this contribution, we aim to describe the above-mentioned transformations and extensions of their applicability. We complete a survey of intrinsic and extrinsic rotations and their mutual transformations.

There are primarily two possible cases for determining the position of a large workpiece in heavy-machine tools.

- The coordinate system of the workpiece is aligned to the position corresponding to the coordinate system of the machine tool and, subsequently, the workpiece is fixed-clamped
- The workpiece is clamped in an unknown position, and the coordinate system of the machine tool is subsequently rotated according to the coordinate system of the workpiece.

The workpiece coordinate system is adapted to the machine coordinate system.

The workpiece is freely positioned in the working space of the machine and is then aligned either manually or automatically (in the case of the use of sophisticated jigs) to a position where the workpiece axes are parallel to the machine axes. Subsequently, the workpiece is clamped and its position is checked again in the event of deformation or other movement. A typical example can be, for example, turbine rotors or generators weighing up to several hundred tons; see Figure 1. Gas turbine machining. This offers the possibility of full automation of measurements and, at the same time, full automation of compensation. The position of such a rotor is measured, for example, by means of a workpiece measuring probe in the basic coordinate system of the machine, where the difference between two points of known distance, in the vertical and horizontal directions of contact, is determined. It is also necessary to consider the bending of the workpiece, which is caused by its own weight. After alignment, the zero point of the workpiece, usually located in the workpiece axis at one of its ends, follows.

Figure 1. Gas turbine machining (formerly operated by Alstom INC and located in Tennessee, USA).

The machine coordinate system is adapted to the workpiece coordinate system.

The workpiece is clamped to the machine working space in an indefinite position, respectively. Repeatability of its clamping is not guaranteed due to dimensions, unworked surfaces, deviating dimensions, etc. A typical example can be a hollow workpiece of cylindrical or ellipsoidal shape, where the outer dimensional surfaces are made in deviations of units of millimeters. Still, the distance between the two fixtures is several tens of meters. The free ends of the workpiece, which are machined, can then be several hundred millimeters, and the inclination in the order of units of degrees relative to all the vectors of the Cartesian coordinate system of the machine, even with a seemingly identical clamping position. To determine the position of such a workpiece, relatively expensive static or mobile scanning systems with long measurement times and output results with an accuracy of 0.005 mm and 0.002 ° can be used, or a workpiece probe provided the workpiece is delivered to the machine with a pre-roughed end/flange from previous production process and can be used as a reference plane. The measurement of the inclination of such a flange is performed in the basic coordinate system of the machine, i.e., its three linear non-tilted axes will suffice. For this purpose, two measuring points are used on the –X-axis and two on the Y-axis with known distances read from the machine measuring system. Standard measuring cycles can be used in manual or automatic machine cycles; see Figure 2: Wind turbine measurement. The resulting generated values of the two angles are used for a transformation cycle that places the tool axis, namely the workpiece probes, perpendicular to the surface of the inclined plane. In this plane, you can focus the rotation of the workpiece around the newly created geometric axis Z, assuming the existing datums. Suppose that these points cannot be unambiguously determined. In that case, a spindle-clamped template is offered instead of a tool that reveals the desired state of a previously known contour (depending on the required / acceptable tolerance). This results in values of three angles, which are then used as input parameters to a transformation cycle that recalculates the relationship between rotations.

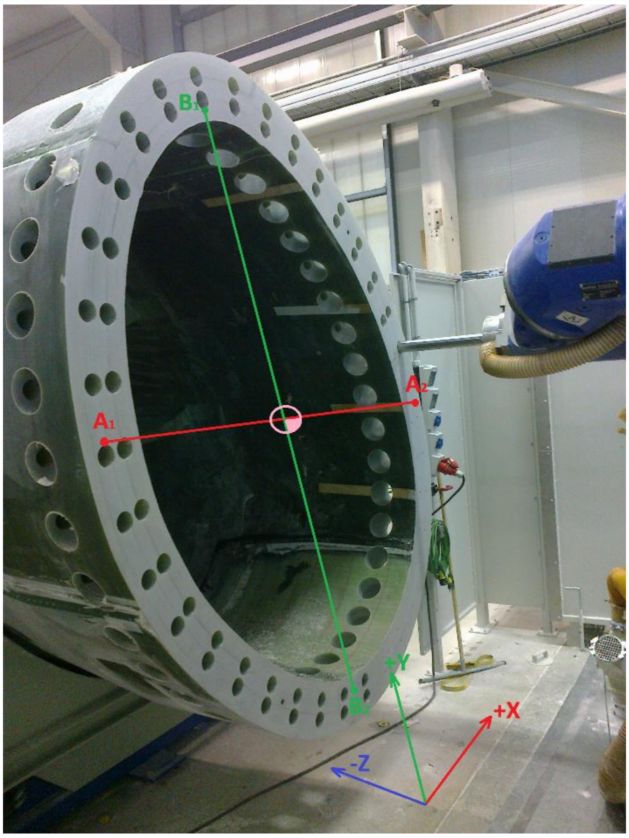

Figure 2. Wind turbine measurement (formerly operated by Enercon GmbH and located in Aurich, Germany).

2 PRACTICAL EXAMPLE

Transformation matrices are used to mathematically express the previous cases. They generally realize three rotations of a solid body and at the same time its displacement (translation).

Transform using displacement

Displacement is defined in such a way that each given point of the object is displayed at a new point displaced by the same vector, thus expressing the displacement of the entire body. The displacement operation is realized by multiplying the expanded coordinate vector by a translation matrix of the form:

$$\begin{pmatrix} B_X \\ B_Y \\ B_Z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & V_x \\ 0 & 1 & 0 & V_Y \\ 0 & 0 & 1 & V_Z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A_X \\ A_Y \\ A_Z \\ 1 \end{pmatrix}, \qquad (1)$$

where Ax, Ay, and Az represent the coordinates of the original point, and Bx, By, and Bz represent the coordinates of the point after displacement. The expansion of matrices and vectors by the value 1 in the last row is introduced here artificially and leads to a natural translation condition:

$$\begin{pmatrix} B_X \\ B_Y \\ B_Z \end{pmatrix} = \begin{pmatrix} A_X \\ A_Y \\ A_Z \end{pmatrix} + \begin{pmatrix} V_X \\ V_Y \\ V_Z \end{pmatrix}. \qquad (2)$$

Transform using rotation

A rotation is defined as turning an object or coordinate system by a given angle about a fixed point (applies in a plane) or an axis in the case of space.

An example of the assembly of rotary matrices is shown in Figure 3. Transform using rotation.

The resulting rotation matrix is given by the product of the submatrices listed above in a specific order. Unlike translation matrices, the order of the matrices in the product matters since the product of matrices is known to be noncommutative.

Example (ZYX rotation) 1

If, for example, a rotation is performed first around the Z-axis (by angle γ), then around the Y-axis (by angle β), and finally around the X-axis (by angle α), the total rotation matrix Rc is given by the product:

$$R_C = R_{ZYX} = R_X \cdot R_Y \cdot R_Z \,. \qquad (3)$$

Individual rotation matrices are placed from right to left and the total rotation matrix Rc represents their product. The matrices are given as:

$$R_X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$R_Y = \begin{pmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (4)$$

$$R_Z = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

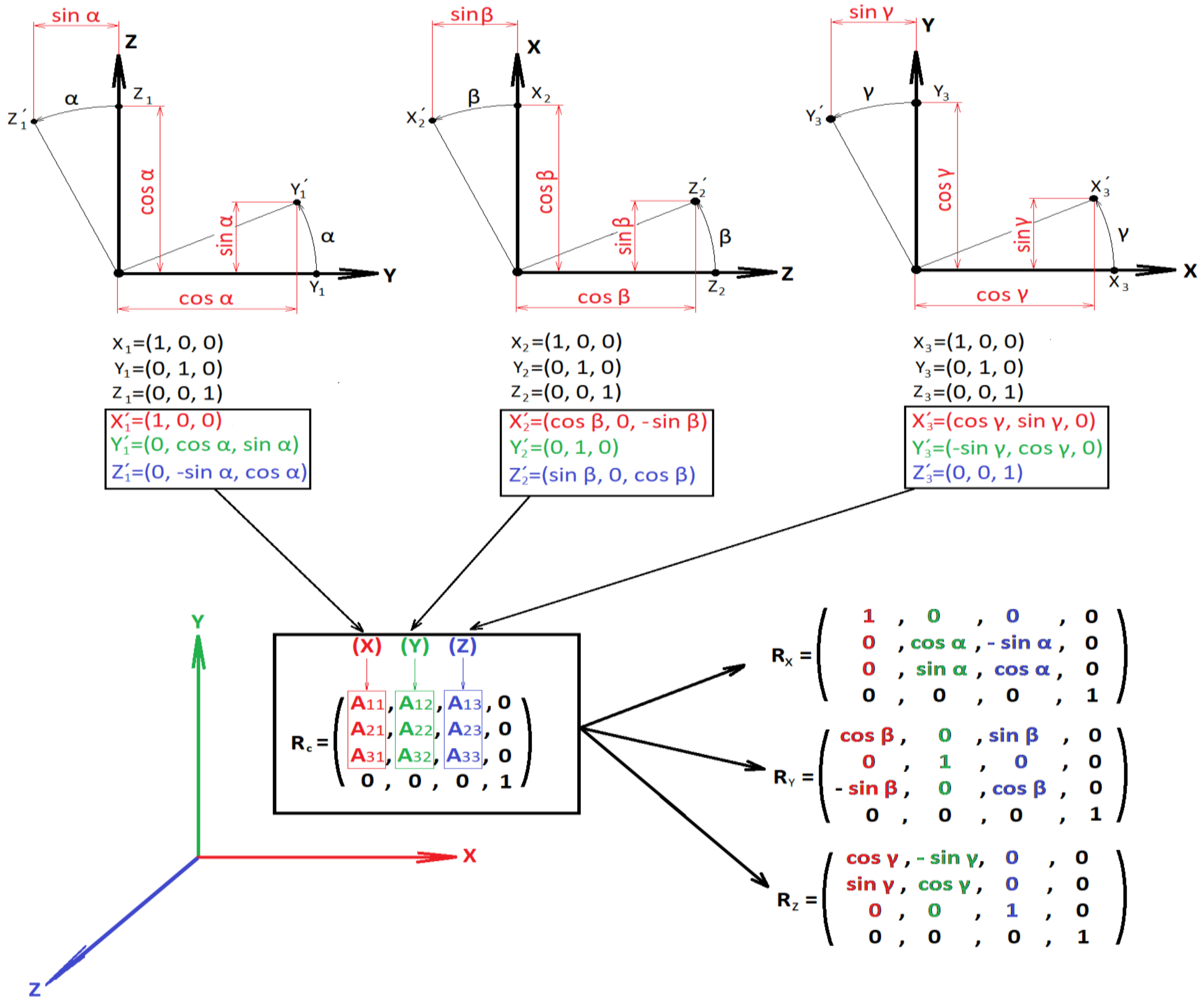

Figure 3. Transform using rotation.

It can be multiplied further. For this, it is convenient to denote *sin* and *cos* of the entering angles as coefficients, so we define:

$$\cos\alpha = c_1,\quad \cos\beta = c_2,\quad \cos\gamma = c_3,$$
$$\sin\alpha = s_1,\quad \sin\beta = s_2,\quad \sin\gamma = s_3. \tag{5}$$

With the help of this notation, the notation of the total rotation matrix after multiplication is simplified to the form:

$$R_{ZYX} = \begin{pmatrix} c_3 c_2 & -c_2\, s_3 & s_2 & 0 \\ c_1\, s_3 + c_3\, s_1\, s_2 & c_1\, c_3 - s_1\, s_2\, s_3 & -c_2\, s_1 & 0 \\ s_1\, s_3 - c_1\, c_3\, s_2 & c_3\, s_1 + c_1\, s_3\, s_2 & c_1\, c_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

Formulas for recalculating angles can be expressed using the positions of individual elements in the matrix A=R_{ZYX}:

$$X(\alpha) = -atan\left(\frac{A_{23}}{A_{33}}\right),$$
$$Y(\beta) = arcsin(A_{13}), \tag{7}$$
$$Z(\gamma) = -atan\left(\frac{A_{12}}{A_{11}}\right).$$

This example describes the resulting rotation matrix, which is built up successively from partial rotations around the axes connected to the original coordinate system (always meaning the individual axes of the machine coordinate system). The same rotation matrix can be constructed equivalently from partial rotations around the axes connected to the rotating body (always meaning a gradual rotation of axis by axis, that is, individual rotations of the system of geometric axes). The first variant of the axes connected to the original coordinate system is called extrinsic rotation, whereas the second variant of the axes connected to the rotating body is called intrinsic rotation. The essential difference when compiling the resulting rotation matrix is the individual order of the subrotation matrices entering the product and applies [Shoemake 1985]:

- with extrinsic rotations, the matrix is multiplied from right to left, and the angles are marked α, β, γ
- with intrinsic rotations, the matrices are multiplied from left to right, and the angles are marked α´, β´, γ´.

If an intrinsic rotation is performed first around the X-axis (by angle α´), then around the Y-axis (by angle β´) and finally around the Z-axis (by angle γ´), then the total rotation matrix Rc´ is given by the product [Shoemake 1985]:

$$R_{C'} = R_{X'Y'Z'} = R_{X'} . R_{Y'} . R_{Z'}. \tag{8}$$

Individual rotation matrices are placed from left to right.

$$R_{X'Y'Z'} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha' & -\sin\alpha' & 0 \\ 0 & \sin\alpha' & \cos\alpha' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\beta' & 0 & \sin\beta' & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta' & 0 & \cos\beta' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\gamma' & -\sin\gamma' & 0 & 0 \\ \sin\gamma' & \cos\gamma' & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

Using a similar notation:

$$\cos\alpha' = c_1',\ \cos\beta' = c_2',\ \cos\gamma' = c_3', \qquad (10)$$
$$\sin\alpha' = s_1',\ \sin\beta' = s_2',\ \sin\gamma' = s_3'.$$

The total intrinsic rotation matrix is simplified to the following form:

$$R_{X'Y'Z'} = \begin{pmatrix} c_2' c_3' & -c_2' s_3' & s_2' & 0 \\ c_3' s_1' s_2' + c_1' s_3' & c_1' c_3' - s_1' s_2' s_3' & -c_2' s_1' & 0 \\ s_1' s_3' - c_1' c_3' s_2' & c_1' s_2' s_3' + c_3' s_1' & c_1' c_2' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (11)$$

Formulas for recalculating angles can also be expressed by using the positions of individual elements in the matrix $A = R_{X'Y'Z'}$:

$$\alpha' = -atan\left(\frac{A_{23}}{A_{33}}\right),$$
$$\beta' = arcsin(A_{13}), \qquad (12)$$
$$\gamma' = -atan\left(\frac{A_{12}}{A_{11}}\right).$$

Relations between Rotations

Any extrinsic rotation is equivalent to an intrinsic rotation by the same angles, but with inverted order of elemental rotations and vice versa [Shoemake 1985]. Based on this definition, the angles for extrinsic rotations and the angles for intrinsic rotations can be expressed from the resulting rotation matrix [Shoemake 1985].

$$R_{ZYX} = R_{X'Y'Z'} \qquad (13)$$

Key formulas

The formulas for calculating α´, β´, γ´ and α, β, γ must also be expressed from the resulting rotation matrix as their opposite permutation for the given rotation sequence:

X-Y-Z: R_C': R_C:

$$\alpha' = -atan\left(\frac{A_{23}}{A_{33}}\right), \quad \alpha = atan\left(\frac{A_{32}}{A_{33}}\right), \qquad (14)$$
$$\beta' = arcsin(A_{13}), \quad \beta = -arcsin(A_{31}),$$
$$\gamma' = -atan\left(\frac{A_{12}}{A_{11}}\right), \quad \gamma = atan\left(\frac{A_{21}}{A_{11}}\right),$$

X-Z-Y: R_C': R_C:

$$\alpha' = atan\left(\frac{A_{32}}{A_{22}}\right), \quad \alpha = -atan\left(\frac{A_{23}}{A_{22}}\right),$$
$$\beta' = atan\left(\frac{A_{13}}{A_{11}}\right), \quad \beta = -atan\left(\frac{A_{31}}{A_{11}}\right), \qquad (15)$$
$$\gamma' = -arcsin(A_{12}), \quad \gamma = arcsin(A_{21}),$$

Y-Z-X: R_C': R_C:

$$\alpha' = -atan\left(\frac{A_{23}}{A_{22}}\right), \quad \alpha = atan\left(\frac{A_{32}}{A_{22}}\right),$$
$$\beta' = -atan\left(\frac{A_{31}}{A_{11}}\right), \quad \beta = atan\left(\frac{A_{13}}{A_{11}}\right), \qquad (16)$$
$$\gamma' = arcsin(A_{21}), \quad \gamma = -arcsin(A_{12}),$$

Y-X-Z: R_C': R_C:

$$\alpha' = -arcsin(A_{23}), \quad \alpha = arcsin(A_{32}),$$
$$\beta' = atan\left(\frac{A_{13}}{A_{33}}\right), \quad \beta = -atan\left(\frac{A_{31}}{A_{33}}\right), \qquad (17)$$
$$\gamma' = atan\left(\frac{A_{21}}{A_{22}}\right), \quad \gamma = -atan\left(\frac{A_{12}}{A_{22}}\right),$$

Z-Y-X: R_C': R_C:

$$\alpha' = atan\left(\frac{A_{32}}{A_{33}}\right), \quad \alpha = -atan\left(\frac{A_{23}}{A_{33}}\right),$$
$$\beta' = -arcsin(A_{31}), \quad \beta = arcsin(A_{13}), \qquad (18)$$
$$\gamma' = atan\left(\frac{A_{21}}{A_{11}}\right), \quad \gamma = -atan\left(\frac{A_{12}}{A_{11}}\right),$$

Z-X-Y: R_C': R_C:

$$\alpha' = arcsin(A_{32}), \quad \alpha = -arcsin(A_{23}),$$
$$\beta' = -atan\left(\frac{A_{31}}{A_{33}}\right), \quad \beta = atan\left(\frac{A_{13}}{A_{33}}\right), \qquad (19)$$
$$\gamma' = -atan\left(\frac{A_{12}}{A_{22}}\right), \quad \gamma = atan\left(\frac{A_{21}}{A_{22}}\right).$$

According to this statement, from one of the resulting rotation matrices it is possible to use these formulas to express the original rotation angles or the angles for another rotation.

Example 2

The successive extrinsic rotations X-Y-Z are given in the following order:

α = 30° around the X axis
β = 45° around the Y axis
γ = 60° around the Z axis

and correspond to the matrix:

$$R_{XYZ} = \begin{pmatrix} 0{,}3536 & -0{,}5732 & 0{,}7392 & 0 \\ 0{,}6124 & 0{,}7392 & 0{,}2803 & 0 \\ -0{,}7071 & 0{,}3536 & 0{,}6124 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (20)$$

Using the formulas for the intrinsic matrix with inverted order of elemental rotations, the angles can be expressed as follows:

$$\alpha' = -atan\left(\frac{A_{23}}{A_{33}}\right) = -atan\left(\frac{0{,}2803}{0{,}6124}\right) \cong -24{,}5939°,$$

$$\beta' = arcsin(A_{13}) = arcsin(0{,}7392) \cong 47{,}6633°, \quad (21)$$

$$\gamma' = -atan\left(\frac{A_{12}}{A_{11}}\right) = -atan\left(\frac{-0{,}5732}{0{,}3536}\right) \cong 58{,}3301°.$$

The resulting rotation matrix for intrinsic rotations, which is also constructed by permutation of $R_{X'Y'Z'}$, (α=-24.5939° around the X axis, β=47.6633° around the Y axis and γ=58.3301° around the Z axis), looks as follows:

$$R_{X'Y'Z'} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & cos -24{,}5939° & -sin -24{,}5939° & 0 \\ 0 & sin -24{,}5939° & cos -24{,}5939° & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} cos\, 47{,}6633° & 0 & sin\, 47{,}6633° & 0 \\ 0 & 1 & 0 & 0 \\ -sin\, 47{,}6633° & 0 & cos\, 47{,}6633° & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} cos\, 58{,}3301° & -sin\, 58{,}3301° & 0 & 0 \\ sin\, 58{,}3301° & cos\, 58{,}3301° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (22)$$

So, the product of the above matrices reads as follows:

$$R_{X'Y'Z'} = \begin{pmatrix} 0{,}3536 & -0{,}5732 & 0{,}7392 & 0 \\ 0{,}6124 & 0{,}7392 & 0{,}2803 & 0 \\ -0{,}7071 & 0{,}3536 & 0{,}6124 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (23)$$

Then we have the following:

$$R_{XYZ}(30{,}45{,}60) = R_{X'Y'Z'}(-24{,}5939; 47{,}6633; 58{,}3301)\,. \quad (24)$$

Assignments of signs

The formulas for calculating α´, β´, γ´ and α, β, γ must also be expressed from the resulting rotation matrix as their opposite permutation for the given rotation sequence.

Looking at the individual formulas, an elegant solution is offered, where certain symmetries are already visible at first glance between the individual formulas:

- when changing the intrinsic and extrinsic rotations, where the arcsine function occurs, there is always a change of sign and at the same time an opposite change of the index of the element in the matrix (e.g. for the Z-X-Y permutation):

$$X(\alpha') = +arcsin(A_{32}) \Rightarrow X(\alpha) = -arcsin(A_{23}). \quad (25)$$

- when changing the intrinsic and extrinsic rotations, where the arctangent function occurs, the sign also changes, and the opposite change in the index of the matrix element occurs only in the numerator:

$$Y(\beta') = -atan\left(\frac{A_{31}}{A_{33}}\right) \Rightarrow Y(\beta) = +atan\left(\frac{A_{13}}{A_{33}}\right),$$

$$Z(\gamma') = -atan\left(\frac{A_{12}}{A_{22}}\right) \Rightarrow Z(\gamma) = +atan\left(\frac{A_{21}}{A_{22}}\right). \quad (26)$$

- for a given intrinsic or extrinsic rotation, the arcsine function always has the opposite sign than the other two arctangent functions :

$$X(\alpha') = +arcsin(A_{32}), \qquad X(\alpha) = -arcsin(A_{23}),$$

$$Y(\beta') = -atan\left(\frac{A_{31}}{A_{33}}\right), \qquad Y(\beta) = +atan\left(\frac{A_{13}}{A_{33}}\right), \quad (27)$$

$$Z(\gamma') = -atan\left(\frac{A_{12}}{A_{22}}\right), \qquad Z(\gamma) = +atan\left(\frac{A_{21}}{A_{22}}\right).$$

If the individual axes of rotation X, Y, and Z are marked with their numerical ordinal index 1, 2, 3 (when X = 1, Y = 2, and Z=3), then additional symmetry elements can be revealed in the relevant matrix. For example, for a given extrinsic rotation permutation Z-X-Y, the middle element (here X, that is, angle α) can only be expressed in the overall matrix using the arcsine function:

$$\begin{pmatrix} 3 & 1 & 2 \\ Z & X & Y \end{pmatrix} = \begin{pmatrix} c_3 c_2 + s_1 s_2 s_3 & -c_2 s_3 + c_3 s_1 s_2 & c_1 s_2 & 0 \\ c_1 s_3 & c_1 c_3 & -s_1 & 0 \\ c_2 s_1 s_3 - c_3 s_2 & c_3 c_2 s_1 + s_3 s_2 & c_2 c_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (28)$$

The sign of the arcsine function can be determined using the determinant of the binary matrix that is constructed for the given rotation permutations. For a general matrix *n x n*, the determinant is given by the Leibniz equation [Brannon 2002]:

$$det\, A = \sum_{\sigma \in S_n} sgn(\sigma) \prod_{i=1}^{n} A_{i,\sigma(i)}, \quad (29)$$

where the sum counts all permutations of the numbers σ {1, 2, ...,n} and sgn(σ) denotes the function of the sign of the permutation. If σ > 0, it is an even permutation, if, on the other hand, σ < 0, it is an odd permutation. In this text, we deal mainly with 3x3 matrices, where the determinant can be determined using the Sarrus rule [Thonton 2004]. Individual axes of rotation are written vertically in the matrix, and index 1 indicates the given axis of rotation, when: R_X = 100, R_Y = 010, R_Z=001. As an example, we determine the sign for extrinsic rotation R_{ZXY}:

$$det\, R_{ZXY} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \Rightarrow det\, R_{ZXY} = 1\,. \quad (30)$$

If the determinant of the extrinsic rotation matrix for the given permutation is positive e.g. $det\, R_{ZXY}$ = 1, then it is an even parity permutation and the arcsine function is also positive, if the determinant of the extrinsic rotation matrix for the given permutation is negative e.g. $det\, R_{YXZ}$ = -1, then it is an odd parity permutation and the arcsine function is also negative [Ciarlet 1996].

$$det\ R_{YXZ} = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow det\ R_{ZXY} = -1 \quad (31)$$

For intrinsic rotations, the meaning of notation is the same, only the resulting determinants have the opposite meaning, i.e. a positive determinant means a negative arcsine function:

$$det\ R_{Z'X'Y'} = 1 \Rightarrow \alpha' = -arcsin(A_{32}),$$
$$det\ R_{ZXY} = 1 \Rightarrow \alpha = arcsin(A_{23}). \quad (32)$$

The entire method of formulating compounds can be expressed graphically; for a better understanding, see Figure 4. Method of formulas compounding.

Additional rotations

There are cases where the coordinate system has already been rotated and it is necessary to apply an additional rotation to the existing one. The same rules as described above apply to these additional rotations:

- with extrinsic rotations, the original matrix of the rotation is with an additional matrix multiplied from right to left

- with intrinsic rotations, the original matrix of the rotation is with an additional matrix multiplied from left to right

The example can be explained by applying an additional intrinsic rotation around the X and Y axes to the original rotation (R_O): The same example is given by applying an additional extrinsic rotation around the X and Y axes to the original rotation (R_O):

$$R_{C'} = R_Y \,.\, R_X \,.\, R_O \,. \quad (33)$$

Composite Transformation of the Space

With a composite transformation of the space from rotation and displacement, the resulting transformation matrix T_c looks like this (an example for intrinsic rotation $R_{Z'Y'X'}$):

$$T_c = R_{Z'Y'X'} \cdot P_c \,. \quad (34)$$

When rotating (in the plane) outside the origin of the basic coordinate system, it is necessary to move the body to the origin, then rotate around the given point and return to the original position. The transformation matrix looks like this:

$$T_c = \begin{pmatrix} 1 & 0 & 0 & \Delta_X \\ 0 & 1 & 0 & \Delta_Y \\ 0 & 0 & 1 & \Delta_Z \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

$$\begin{pmatrix} c_3'c_2' & -c_1's_3' + c_3'\,s_2'\,s_1' & s_1'\,s_3' + c_1'\,c_3'\,s_2' & 0 \\ c_2'\,s_3' & c_1'c_3' + s_1'\,s_2'\,s_3' & -c_3'\,s_1' + c_1'\,s_3'\,s_2' & 0 \\ -s_2' & c_2'\,s_1' & c_1'\,c_2' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \quad (35)$$

$$\begin{pmatrix} 1 & 0 & 0 & -\Delta_X \\ 0 & 1 & 0 & -\Delta_Y \\ 0 & 0 & 1 & -\Delta_Z \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where: Δ_X, Δ_Y, Δ_Z means displacement to the origin of the coordinate system and $-\Delta_X$, $-\Delta_Y, -\Delta_Z$ means displacement back to the origin point.

3 APPLICATION: CYCLE TO RECALCULATE ROTATIONS

When modern measurement methods with workpiece probes and standard supplied cycles are used, in some cases, an erroneous evaluation of the measurement results occurs. A typical example of such an incorrect evaluation of measurement results can be the machining of a wind turbine blade when the blade body is so long that the workpiece cannot be set parallel to the machine axes. The workpiece probe can measure A1-A2 and B1-B2 and determine rotations around the Y and X axes; see Figure 2. Wind turbine measurement.

However, the rotation angles obtained this way cannot be entered directly into the standard CYCLE800 cycle, as the angles are associated with rotation around the machine coordinate system. On the contrary, the angles related to rotation around the workpiece coordinate system are used to enter CYCLE800.

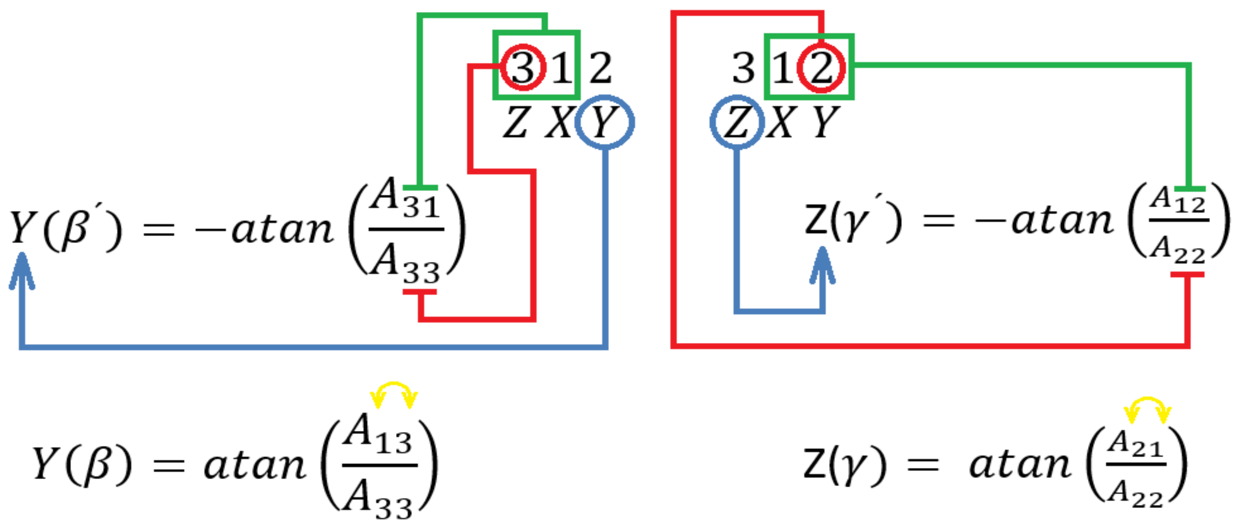

Figure 4. Method of formulas compounding.

Another reason may be that, when using the standard transformation cycle CYCLE800, the number of rotation axes of the kinematic chain is limited to one or, at most, two rotary axes. Therefore, this cannot be used for the standard configuration when the machine is equipped with a rotary or tilting rotary table and, simultaneously, an angular milling head with two axes of rotation, see Figure 5. Milling head IFVW207.

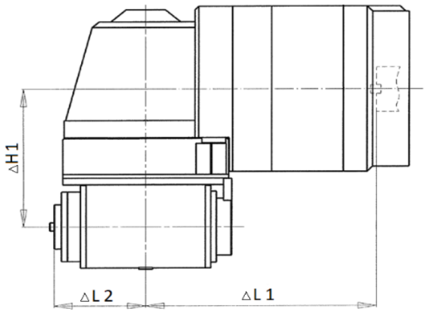

Figure 5. Milling head IFVW207.

For that reason, it was necessary to develop a new cycle, with the help of which the required transformation in the coordinate system and the physical rotation of the necessary axes will take place. Based on the relationships and formulas derived, a CYCLE600 cycle was created to recalculate the intrinsic and extrinsic rotations.

The basic prerequisite for the functionality of the CYCLE600 is that the zero point of the machine is identical to the center of the rotary table, which is in the zero position. That is, the X machine axis is equal to the center of the table, the Y machine axis is identical to the edge of the table clamping rotary plate, the Z machine axis is equal to the table rotation axis, see Figure 6. Definition of the zero points.

In the case of heavy horizontal machine tools where a quill, spindle, tilt table, and indexing head configuration with two rotation axes is used, the Z, V and W axes are parallel, and it is necessary to convert the zero point of the workpiece to the Z axis before using CYCLE600.

The actual invocation of CYCLE600 looks like this:

CYCLE600 (_INEXIN, _MIHEAD, _SW_REC, _BIN_IN, _POS_X, _POS_Y, _POS_Z, _ANG_1, _ANG_2, ANG_3, _POS_X_R, _POS_Y_R, _POS_Z_R, _INEXOUT, _BIN_OUT, _ROT_AD, _ROT_INV, _RETR_Z)

_INEXIN	*Bool*
_MIHEAD	*String*
_SW_REC	*Bool*
_BIN_IN	*Integer*
_POS_X	*Real*
_POS_Y	*Real*
_POS_Z	*Real*
_ANG_1	*Real*
_ANG_2	*Real*
_ANG_3	*Real*
_POS_X_R	*Real*
_POS_Y_R	*Real*
_POS_Z_R	*Real*
_INEXTOUT	*Bool*
_BIN_OUT	*Integer*
_ROT_AD	*Bool*
_ROT_INV	*Bool*
_RETR_Z	*Bool*

Explanation of Local Parameters

_INEXIN - Input choice of rotation: (1) intrinsic rotation; (0) extrinsic rotation

_MIHEAD - Name of the selected technological accessory to be used in CYCLE600, for example, “IFVW207”. The selected accessory is checked against the currently used accessory on the machine, and at the same time, the parameters are read from the HEAD_DEF cycle. In the event of a mismatch (e.g., accessory is not defined), an error message is displayed

_SW_REC - Choice of recalculation only (1), can be used after the restart of the machine or to load the calculated parameters in GUD (Global User Data); (0) rotation with the accessory or rotary table

_BIN_IN - Input binary combination of coordinate system. A binary code is assigned to each axis (X=01, Y=10, Z=11), and the selection of the rotation permutation creates the corresponding rotation in decimal code. According to the parameter *_INEXIN*, the loading order is determined (from left to right for the intrinsic rotation or right to left for the extrinsic rotation). Extrinsic rotation: Z-X-Y =100111 = 39, X-Y-Z = 111001 = 57, intrinsic rotation: Z′-X′-Y′ = 110110 = 57, X′-Y′-Z′ = 011011 = 39

_POS_X - translation of the reference point in the X – axis before rotation

_POS_Y - Translation of the reference point in the Y – axis before rotation

_POS_Z - translation of the reference point in the Z – axis before rotation

_ANG_1 - Rotation around the first axis

_ANG_2 - Rotation around the second axis

_ANG_3 - Rotation around the third axis

_POS_X_R - translation of the reference point in the X-axis after rotation.

_POS_Y_R - translation of the reference point in the Y – axis after rotation

_POS_Z_R - Translation of the reference point in Z – axis after rotation

_INEXOUT - Output choice of rotation: (1) intrinsic rotation; (0) extrinsic rotation

_BIN_OUT - Output binary combination of coordinate system. A binary code is assigned to each axis (X=01, Y=10, Z=11), and the selection of the rotation permutation creates the corresponding rotation in decimal code. According to the parameter *_INEXOUT*, the loading order is determined (from left to right for the extrinsic rotation or from right to left for the intrinsic rotation). Extrinsic rotation: Z-X-Y = 111001 = 57, X-Y-Z = 100111 = 39, intrinsic rotation: Z′-X′-Y′ = 100111 = 39, X′-Y′-Z′ = 111001 = 57

_ROT_AD - Choice of new output rotation (0) with the accessory or rotary table; (1) additive rotation to the last used one with the accessory or rotary table

_ROT_INV – For accessories with two axes of rotation, the same position can be achieved by an inverse rotation of 180 degrees in both axes. (0) to use the calculated position of the accessory; (1) to use the inverse position of the accessory

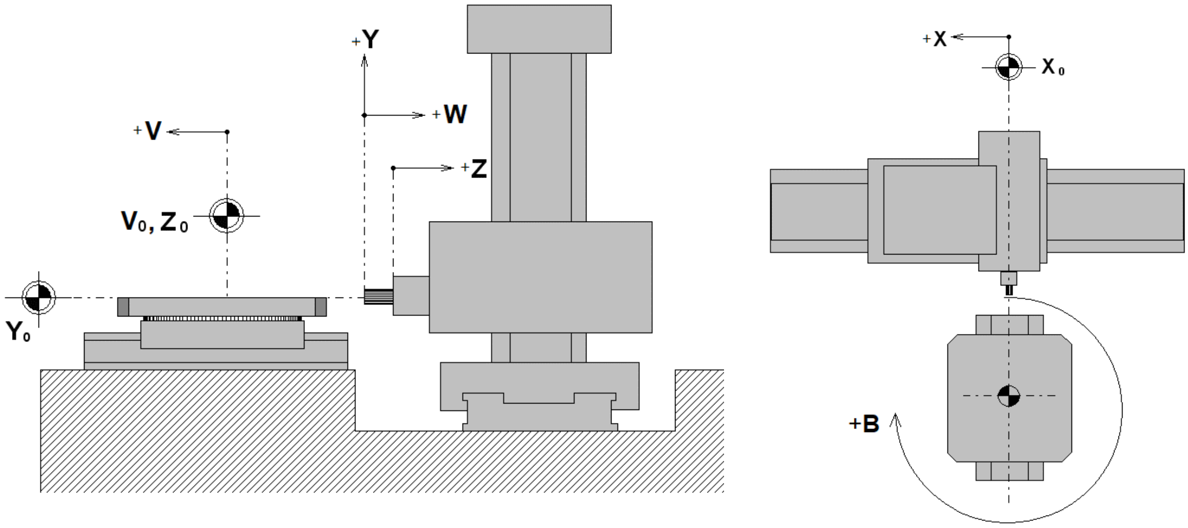

Figure 6. Definition of the zero points.

_RETR_Z – Retract with the tool prior to the rotation (1) with the accessory or rotary table; or (0) not to retract with the tool

Explanation of Global Parameters

Passing parameters between different cycles happens via global variables, defined in the Sinumerik system as GUD. These parameters remain in the memory even after the machine is turned off. For these reasons, MGUD_CYC600.DEF was created for CYCLE600, where variables are defined that can then be used in other cycles.

An example would be to convert extrinsic rotation to intrinsic rotation and use these parameters to invoke the standard cycle CYCLE800.

_MIHEAD	*_CYC600_1*
_ANG_1_OUT	_CYC600_2
_ANG_2_OUT	_CYC600_3
_ANG_3_OUT	_CYC600_4
_INEXTOUT	_CYC600_5
_BIN_OUT	_CYC600_6

Sub-cycle HEAD_DEF

The *HEAD_DEF* cycle defines individual technological accessories, including length and offset parameters. The rotary table is, in this case, meant as a technological accessory.

Milling heads are defined in the HEAD_DEF cycle in this form:

```
N830    TZ7:
N840      _HEAD_NAME="IFVW207S"
N850      _HEAD_ALLOWED=1
N860      _DELTA_L1=540.735 _DELTA_L2=250.049
N880      _DELTA_H1=200.007
N880      _OFFS_L1_X=0.024 _OFFS_L1_Y=0.032
N890      _OFFS_L2_X=0.011 _OFFS_L2_Y=-0.041
N900      _OFFS_H1_X=0.335 _OFFS_H1_Z=0.247
N910     GOTOF END.
```

Example 3

The flange, see Fig. 2 Wind turbine measurement, was measured with a workpiece measuring probe and calculated rotation of the coordinate system rot X = 30 degrees, rot Y = 45 degrees, and rot Z = 60 degrees. In this case, it is a rotation of the machine's coordinate system, so it is an extrinsic rotation R_{XYZ}. The UFK600 technological accessory was selected for the machining of this flange, and the rotation of the space in order Z-X-Y was selected. To reach the position, it is necessary to rotate the technological accessory around the workpiece coordinate system, so it is an intrinsic rotation $R_{Z'X'Y'}$.

1 - the example could be to convert extrinsic rotations to intrinsic rotations and to use these parameters to invoke the standard cycle CYCLE800.

_INEXIN	*0*
_MIHEAD	*“UFK600”*
_SW_REC	*1*
_BIN_IN	*57*
_POS_X	*0*
_POS_Y	*0*
_POS_Z	*0*
_ANG_1	*30*
_ANG_2	*45*
_ANG_3	*60*
_POS_X_R	*0*
_POS_Y_R	*0*
_POS_Z_R	*0*
_INEXTOUT	*0*
_BIN_OUT	*39*
_ROT_AD	*0*
_ROT_INV	*0*
_RETR_Z	*0*

N120 CYCLE600 (0, “UFK600“, 1, 57, 0, 0, 0, 30, 45, 60, 0, 0, 0, 0, 1, 39, 0, 0, 0)

N125 STOPRE

```
N130 IF _CYC600_5==1
N135 CYCLE800(0, _CYC600_1, 0, _CYC600_6, 0,0,0, _CYC600_2, _CYC600_3, _CYC600_4,0,0,0,-1)
N140 ENDIF
```

Invoking CYCLE600 saves the following results in M_GUD_CYC600:

_CYC600_1	*"UFK600"*
_CYC600_2	*37.7923*
_CYC600_3	*20.7048*
_CYC600_4	*49.1066*
_CYC600_5	*1*
_CYC600_6	*39*

2 - the example could be to convert extrinsic rotations to other extrinsic rotations and using these parameters to invoke the standard cycle CYCLE800.

_INEXIN	*0*
_MIHEAD	*"UFK600"*
_SW_REC	*1*
_BIN_IN	*57*
_POS_X	*0*
_POS_Y	*0*
_POS_Z	*0*
_ANG_1	*30*
_ANG_2	*45*
_ANG_3	*60*
_POS_X_R	*0*
_POS_Y_R	*0*
_POS_Z_R	*0*
_INEXTOUT	*1*
_BIN_OUT	*39*
_ROT_AD	*0*
_ROT_INV	*0*
_RETR_Z	*0*

```
N120 CYCLE600 (0, "UFK600", 1, 57, 0, 0, 0, 30, 45, 60, 0, 0, 0, 0, 0, 39, 0, 0, 0)
N125 STOPRE
```

Invoking CYCLE600 saves the following results in M_GUD_CYC600:

_CYC600_1	*"UFK600"*
_CYC600_2	*39.6392*
_CYC600_3	*-16.2799*
_CYC600_4	*50.3607*
_CYC600_5	*1*
_CYC600_6	*39*

4 CONCLUSIONS

The cycle described above was created primarily for easy recalculation of rotations that are measured by using a workpiece probe (extrinsic rotation) and subsequent rotation of the machining tool using a technological accessory (intrinsic rotation).

Without the use of the CYCLE600, the standard procedure was to measure the workpiece position in the working place, send the measured data to the post-processor, generate the program and start the actual machining process. Especially in serial production, it is possible to create only one program for a given type of workpiece and, by using the CYCLE600, call up the measured data as a parameter and use them in this program. In this way, there will be a significant reduction of secondary times when you do not have to wait for a new program to be generated, but you can immediately change the measuring probe for the machining tool and start the machining process.

If there is a demand for the use of the cycle by more customers, future work will focus on creating graphic support directly in the Sin840 base mask and overall user-friendliness. Additionally, create user documentation with a detailed description of individual parameters and graphic processing of specific examples of use.

All calculations described above were checked by using MATLAB software, and at the same time the freely available libraries were extended for this software to recalculate intrinsic and extrinsic rotations for different spin order permutations and are stored for free download at MATLAB Central under the tab "Evaluation of intrinsic rotations" or directly via the link:

https://www.mathworks.com/matlabcentral/fileexchange/133287-evaluation-of-intrinsic-rotations

References

Book:

[Brannon 2002] Brannon M. Rebecca and co-authors. Rotation - A review of useful theorems involving proper orthogonal matrices referenced to three-dimensional physical space. Albuquerque : Sandia National Laboratories, 2002. NM 87185-0820.

[Ciarlet 1996] P.G., Ciarlet and J.L., Lions. Handbook of Numerical Analysis, Vol. 4, Finite Element Methods - Numerical Methods for Solids (Part 2). Elsevier : Elsevier Science, 1996. ISBN 9780444817945.

[Thonton 2004] Stephen, Thonton, T., and Jerry, Marion, B. Classical Dynamics of Particles and Systems, Fifth Edition. Bethlehem, PA : Lehigh Press, 2004. ISBN 0-534-40896-6.

[Shoemake 1985] Ken, Shoemake. Rotation with Quarterion Curves Proc. San Francisco, CA : Sig-Graph, 1985. ACM 0-89791-166-0/85/007/0245.

[Siemens manual 2017] 840D sl. Sinumerik Operate V4.7 5-Axis-Workshop Technology milling. Nürnberg: Siemens AG, 2017.

Paper in Electronic Journal:

[Ohnistova 2018] Ohnistova P., Pista M., Prat D. (2018), Polynomial Interpolation in 5-Axis Milling. MM Science Journal 2018. http://doi.org/10.17973/MMSJ.2018_11_201816

[Kuprin 2022] Kuprin N., Sveda J., Chladek S., Havlan T., Novotny L., Koubek J. (2022), New Device for Rapid Measurement of Machine Tool Geometric Errors 2022. http://doi.org/10.17973/MMSJ.2022_10_2022120

[Holub 2015] Holub M., Blecha P., Bradac F., Kana R. (2015), Volumetric Compensation of Three-Axis Vertical Machining Centre 2015. http://doi.org/10.17973/MMSJ.2015_10_201534

CONTACTS:

Radek Lehner **Ing.**
University of West Bohemia, Faculty of Mechanical Engineering, Department of Machining Technology
Univerzitni 8, Pilsen, 301 00, Czech Republic