DUAL NUMBERS ARITHMETIC IN MULTI AXIS MACHINE ERROR MODELING

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When a kinematic chain of a multiaxis machine centre is assembled by means of homogeneous matrices, it is possible to include the error representing matrices within and neglect the error terms which do not affect the prescribed accuracy. Classically, such error terms are identified and neglected according to the system of given identities after the matrix multiplication. In our approach, the matrices itself are designed to form a ring that respects the desired arithmetic of error terms, particularly the ring of matrices over the dual numbers. On the other hand, to make this algebraically possible, several negligible terms remain.

KEYWORDS

dual number, geometric accuracy, volumetric accuracy, CNC machine tools

1 INTRODUCTION

With increasing demands on manufacturing accuracy of machine tools, there is a proportional need to enhance the geometric and working accuracy. The resulting working accuracy of three-axis CNC machine tool is affected up to 70% by quasi-static errors [Ramesh 2000]. This category of quasi-static errors includes geometric errors, temperature errors and errors caused by deformation of the own weight of machine parts. For a five-axis machine tool, this percentage is even higher [Ibaraki 2012].

In order to enhance the geometric accuracy, new methodological approaches are constantly being developed to describe geometric deviations of the machine tool with the output for software compensations. Such demands are laid not only on the existing technologies of measurement, but also on the development of new measuring equipment and measurement procedures. Already calculated deviations are then implemented to the control systems of machine tools.

Achievement of higher geometric accuracy requires growing demands on accuracy of measuring equipment, methodologies of measurement procedures, and quality of data processing and evaluation [Linares 2014]. However, this also increases the demands on computing equipment in the form of machine tool models to calculate deviations that are sufficiently accurate, fully defined and inexpensive in terms of hardware.

A large part of authors deal with modelling of deviations in machining centres. Most of them then operate with the approach using the homogeneous transformation matrices (HTM) [Rahman 2000, Okafor 2000, Uddin 2009, Tian 2014]. The use of these homogeneous transformation matrices leads to appropriate simplifications. The author [Okafor 2000] e.g. uses the HTM method to calculate deviations, but the resulting relationships were supplemented with his own extensions which do not contain in the aforementioned HTM.

A proper description of deviations in the workspace of the machine tool is necessary for subsequent processing of compensation data. These compensation data then can be used for single-axis compensation, dependent compensations of two axes or volumetric compensations. The aim of numeric compensations is to minimize the real deviation TCP (Tool Center Point) from the desired position of the machine.

This article discusses the possibilities of streamlining the calculation of geometric deviations on three-axis machine tool by HTM using dual numbers [Holub 2015b]. Moreover, it compares the results of standard calculation by HTM and by HTM using dual numbers.

2 GEOMETRIC DEVIATIONS OF THREE-AXIS MACHINE TOOLS

2.1 *Description of geometric deviation*

Description and the number of geometrical deviations of machine tools are generally based on the number of CNC controlled axes and the coordinate system and are defined according to ISO 841. Figure 1 shows the diagram of a three-axis vertical milling machine 21, including all 21 geometric deviations. These are errors of approaching the position in the axis EXX, EYY, EZZ, straightness errors EYX, EZX, EXY, EZY, EXZ, EYZ, angular errors EAX, EBX, ECX, EAY, EBY, ECY, EAZ, EBZ, ECZ and errors of angle A0Z, B0Z, C0Y.

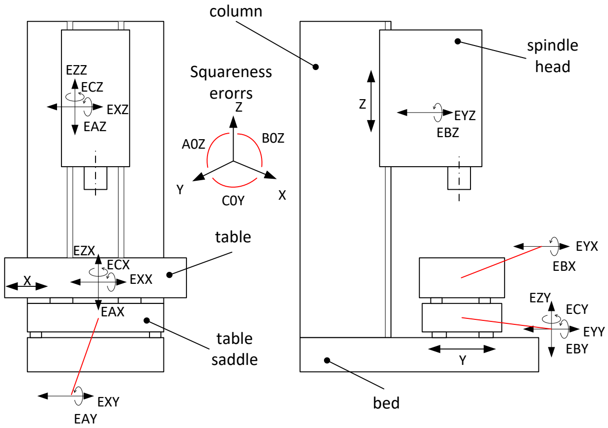

Figure 1. Geometric deviations of three-axis vertical machining centre [Holub 2015a]

To identify the above mentioned deviations, various measuring devices and procedures are used; these can evaluate all 21 geometric deviations simultaneously. These devices include self-tracking laser interferometers LaserTRACER [Holub 2014] or Laser Tracker [Knobloch 2014]. To measure the workspace of the machine tool, these devices use a sequential multilateration principle. This provides a sufficient accuracy of measurement to assess the CNC machine tool. Another group consists of measuring devices which can identify only some of the abovementioned deviations and then it is necessary to use a combination of more than one measuring device and measurement procedures. These devices include single-axis laser interferometers [Marek 2009], digital spirit levels, collimators or control prisms and dial indicators.

2.2 *Measuring data*

All 21 parameters of errors obtained from the measurement with LaserTRACER are shown in the following Table 1. These deviations are then used for following calculations of volumetric errors.

Group	Parameter ISO Norm	Parameter Calculation	Deviation (range)
Position [µm]	EXX	δ_{xx}	5.843679335
	EYY	δ_{yy}	-7.929991163
	EZZ	δ_{zz}	-0.071256382
Straightness [µm]	EYX	δ_{yx}	2.559352218
	EZX	δ_{zx}	0.800863724
	EXY	δ_{xy}	1.549784038
	EZY	δ_{zy}	-0.099355403
	EXZ	δ_{xz}	-1.966586041
	EYZ	δ_{yz}	1.260893805
Pitch / Yaw / Roll [µrad]	EAX	ε_{xx}	-8.998678
	EBX	ε_{yx}	9.180649
	ECX	ε_{zx}	-9.258189
	EAY	ε_{xy}	12.041728
	EBY	ε_{yy}	-5.934292
	ECY	ε_{zy}	-3.52876
	EAZ	ε_{xz}	1.758333
	EBZ	ε_{yz}	48.368974
	ECZ	ε_{zz}	-9.309235
Squareness [µrad]	C0Y	α_{xy}	27.950254
	B0Z	α_{xz}	-73.699513
	A0Z	α_{zy}	32.473779

Table 1. Measuring data

3 MATHEMATICAL BACKROUND

3.1 Preliminaries

Classically, if the volumetric error is handled, the error matrices are used, e.g. the matrix of the translation along the axis error is of the form

$$E_x = \begin{pmatrix} 1 & -\check{n}_{zx} & \check{n}_{yx} & \delta_{xx} \\ \check{n}_{zx} & 1 & -\check{n}_{xx} & \delta_{yx} \\ -\check{n}_{yx} & \check{n}_{xx} & 1 & \delta_{zx} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (1)$$

This can be derived using the Euler matrices expanded into the Taylor series once you neglect the terms in which two or more error terms are multiplied. Indeed,

$$\begin{pmatrix} 1 & 0 & 0 & \delta_{xx} \\ 0 & 1+O(\check{n}^2) & -\check{n}_{xx}+O(\check{n}^2) & 0 \\ 0 & \check{n}_{xx}+O(\check{n}^2) & 1+O(\check{n}^2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1+O(\check{n}^2) & 0 & \check{n}_{yx}+O(\check{n}^2) & 0 \\ 0 & 1 & 0 & \delta_{yx} \\ -\check{n}_{yx}+O(\check{n}^2) & 0 & 1+O(\check{n}^2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1+O(\check{n}^2) & -\check{n}_{zx}+O(\check{n}^2) & 0 & 0 \\ \check{n}_{zx}+O(\check{n}^2) & 1+O(\check{n}^2) & 0 & 0 \\ 0 & 0 & 1 & \delta_{zx} \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$
$$\begin{pmatrix} 1+O(\check{n}^2) & -\check{n}_{zx}+O(\check{n}^2) & \check{n}_{yx}+O(\check{n}^2) & \delta_{xx}+O(\check{n})\delta \\ \check{n}_{zx}+O(\check{n}^2) & 1+O(\check{n}^2) & -\check{n}_{xx}+O(\check{n}^2) & \delta_{yx}+O(\check{n})\delta \\ -\check{n}_{yx}+O(\check{n}^2) & \check{n}_{xx}+O(\check{n}^2) & 1+O(\check{n}^2) & \delta_{zx}+O(\check{n})\delta \\ 0 & 0 & 0 & 1 \end{pmatrix} \to$$
$$\begin{pmatrix} 1 & -\check{n}_{zx} & \check{n}_{yx} & \delta_{xx} \\ \check{n}_{zx} & 1 & -\check{n}_{xx} & \delta_{yx} \\ -\check{n}_{yx} & \check{n}_{xx} & 1 & \delta_{zx} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (2)$$

where $\check{n}$ stands for one of the symbols $\check{n}_{xx}, \check{n}_{zx}, \check{n}_{yx}$ and by δ we understand any symbol of the triple $\delta_{xx}, \delta_{zx}, \delta_{yx}$.

Algebraically, if all functions within the matrices are considered in the form of the Taylor series, the result corresponds to the calculations within the Weyl algebra $\square[\check{n}_{xx}, \check{n}_{yx}, \check{n}_{zx}] / \langle \check{n}^2, \delta^2, \check{n}\delta \rangle$. This structure is rather abstract and thus more convenient setting can be used. As algebraically the properties of $\check{n}_{xx}, \check{n}_{yx}, \check{n}_{zx}$ coincide we can consider these as the coefficient of one element only and thus we can work in the dual numbers algebra, see [Hrdina 2014]. Note that this simplification brings minor complications, particularly the number of remaining terms increases, but, on the other hand, the question of the geometric interpretation of these higher order error terms is also interesting. For instance, one can distinguish the rotation error w.r.t. the x and y axis while translating along the axis x.

Let us recall the definition of the dual numbers. It is a set

$$\mathrm{D} = \{a + b\iota \mid a, b \in \square\} \quad (3)$$

endowed with the operations summation and multiplication

$$\begin{aligned} (a_1 + b_1\iota) + (a_2 + b_2\iota) &= (a_1 + a_2) + (b_1 + b_2)\iota, \\ (a_1 + b_1\iota) \cdot (a_2 + b_2\iota) &= (a_1 a_2) + (a_1 b_2 + b_1 a_2)\iota. \end{aligned} \quad (4)$$

satisfying the identity. Its elements can therefore be understood as the factorized polynomial ring

$$\square[x] / \langle x^2 \rangle \quad (5)$$

Note that even higher order dual numbers can be considered, e.g. the second order dual numbers are defined as the set

$$\mathrm{D}_2 = \{a + b\iota + c\iota^2 \mid a, b, c \in \square\} \quad (6)$$

endowed with the operations summation and multiplication

$$\begin{aligned} (a_1 + b_1\iota + c_1\iota^2) + (a_2 + b_2\iota + c_2\iota^2) &= (a_1 + a_2) + (b_1 + b_2)\iota + (c_1 + c_2)\iota^2, \\ (a_1 + b_1\iota + c_1\iota^2) \cdot (a_2 + b_2\iota + c_2\iota^2) &= (a_1 a_2) + (a_1 b_2 + b_1 a_2)\iota + (a_1 c_2 + b_1 b_2 + c_1 a_2)\iota^2. \end{aligned} \quad (7)$$

In the language of the factorized polynomial rings this corresponds to the set $\square[x] / \langle x^3 \rangle$. Further generalizations contain e.g. $\mathrm{D}_3, \mathrm{D}_4, \ldots$. Further generalizations contain e.g. $\mathrm{D}_3, \mathrm{D}_4, \ldots$.

For calculations, the matrix representation of these structures is quite interesting. Although these representations are not unique, there are some classical choices such as $\iota \mapsto \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$. Indeed $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}^2$ and thus any dual number $a + b\iota \in \mathrm{D}, a, b \in \square$ is represented as 2×2 matrix $\begin{pmatrix} a+b & b \\ -b & a-b \end{pmatrix}$. Even more common choice is the following

$\iota \mapsto \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Indeed $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}^2$ and thus any dual number $a + b\iota \in \mathrm{D}, a, b \in \square$ is represented as 2×2 matrix $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$. Note that the second order dual numbers

$a+b\iota+c\iota^2$ can be represented e.g. by the matrix $\begin{pmatrix} a & b & c \\ 0 & a & b \\ 0 & 0 & a \end{pmatrix}$.

As far as the accuracy questions are involved, if the terms of the micron--level are placed on the secondary diagonal, the by multiplication the term on the ternary diagonal is 10^{-3} micron--level. For a machine with 1 meter in diagonal, if the Abe principle is applied, the secondary diagonal contains millimetre--level terms and the ternary diagonal contains the micron-level terms. But this coincides with the classical approach meaning that no larger terms than 10^{-3} micron-level should be neglected. When considerably larger machines are studied, higher order dual numbers should be employed.

3.2 Error matrices

In the sequel, we use the matrices over second order dual numbers, i.e. the matrix elements will be of the form $a+b\iota+c\iota^2$ and furthermore, we use the dual numbers matrix representation meaning that each element is represented as 3×3 matrix $\begin{pmatrix} a & b & c \\ 0 & a & b \\ 0 & 0 & a \end{pmatrix}$. The proof that such representation is isomorphic to the second order dual numbers is rather straightforward. Consequently, e.g. the rotation matrix w.r.t. the X axis while translating along the X axis is of the form

$$\begin{pmatrix} 1 & 0 & 0 & \delta_{xx}\iota \\ 0 & 1 & -\check{n}_{xx}\iota & 0 \\ 0 & \check{n}_{xx}\iota & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&\delta_{xx}&0 \\ 0&1&0&0&0&0&0&0&0&0&0&\delta_{xx} \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&-\check{n}_{xx}&0&0&0&0 \\ 0&0&0&0&1&0&0&0&-\check{n}_{xx}&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&\check{n}_{xx}&0&1&0&0&0&0&0 \\ 0&0&0&0&0&\check{n}_{xx}&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix} \quad (8)$$

Should the calculations follow the D_2 arithmetic precisely, even the error matrices must be recalculated. Then we obtain

$$\begin{pmatrix} 1 & 0 & 0 & \delta_{xx}\iota \\ 0 & 1 & -\check{n}_{xx}\iota & 0 \\ 0 & \check{n}_{xx}\iota & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \check{n}_{yx}\iota & 0 \\ 0 & 1 & 0 & \delta_{yx}\iota \\ -\check{n}_{yx}\iota & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\check{n}_{zx}\iota & 0 & 0 \\ \check{n}_{zx}\iota & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta_{zx}\iota \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

$$= \begin{pmatrix} 1 & -\check{n}_{zx}\iota & \check{n}_{yx}\iota & \delta_{xx}\iota - \check{n}_{yx}\delta_{zx}\iota^2 \\ \check{n}_{zx}\iota + (\check{n}_{xx}\check{n}_{yx})\iota^2 & 1 & -\check{n}_{xx}\iota & \delta_{yx}\iota - \check{n}_{xx}\delta_{zx}\iota^2 \\ -\check{n}_{yx}\iota + (\check{n}_{xx}\check{n}_{zx})\iota^2 & \check{n}_{xx}\iota + (\check{n}_{yx}\check{n}_{zx})\iota^2 & 1 & \delta_{zx}\iota + \check{n}_{xx}\delta_{yx}\iota^2 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (10)$$

Note that this matrix differs from the classical one exactly in the second order error terms. Yet it is notable that the matrix multiplication in this case is not commutative, indeed when the order is changed the second order error terms are different:

$$\begin{pmatrix} 1 & 0 & \check{n}_{yx}\iota & 0 \\ 0 & 1 & 0 & \delta_{yx}\iota \\ -\check{n}_{yx}\iota & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & \delta_{xx}\iota \\ 0 & 1 & -\check{n}_{xx}\iota & 0 \\ 0 & \check{n}_{xx}\iota & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\check{n}_{zx}\iota & 0 & 0 \\ \check{n}_{zx}\iota & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta_{zx}\iota \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\check{n}_{zx}\iota + (\check{n}_{xx}\check{n}_{yx})\iota^2 & \check{n}_{yx}\iota & \delta_{xx} - \check{n}_{yx}\delta_{zx}\iota^2 \\ \check{n}_{zx}\iota & 1 & -\check{n}_{xx}\iota & \delta_{yx} - \check{n}_{xx}\delta_{zx}\iota^2 \\ -\check{n}_{yx}\iota + (\check{n}_{xx}\check{n}_{zx})\iota^2 & \check{n}_{xx}\iota + (\check{n}_{yx}\check{n}_{zx})\iota^2 & 1 & \delta_{zx} + \check{n}_{yx}\delta_{xx}\iota^2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (11)$$

Nevertheless, this is only natural. Consequently the following error matrices are obtained:

$$E_x := \begin{pmatrix} 1 & -\check{n}_{zx}\iota & \check{n}_{yx}\iota & \delta_{xx}\iota - \check{n}_{yx}\delta_{zx}\iota^2 \\ \check{n}_{zx}\iota + (\check{n}_{xx}\check{n}_{yx})\iota^2 & 1 & -\check{n}_{xx}\iota & \delta_{yx}\iota - \check{n}_{xx}\delta_{zx}\iota^2 \\ -\check{n}_{yx}\iota + (\check{n}_{xx}\check{n}_{zx})\iota^2 & \check{n}_{xx}\iota + (\check{n}_{yx}\check{n}_{zx})\iota^2 & 1 & \delta_{zx}\iota + \check{n}_{xx}\delta_{yx}\iota^2 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (12)$$

$$E_y := \begin{pmatrix} 1 & -\check{n}_{zy}\iota & \check{n}_{yy}\iota & \delta_{xy}\iota - \check{n}_{yy}\delta_{zy}\iota^2 \\ \check{n}_{zy}\iota + (\check{n}_{xy}\check{n}_{yy})\iota^2 & 1 & -\check{n}_{xy}\iota & \delta_{yy}\iota - \check{n}_{xy}\delta_{zy}\iota^2 \\ -\check{n}_{yy}\iota + (\check{n}_{xy}\check{n}_{zy})\iota^2 & \check{n}_{xy}\iota + (\check{n}_{yy}\check{n}_{zy})\iota^2 & 1 & \delta_{zy}\iota + \check{n}_{xy}\delta_{yy}\iota^2 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (13)$$

$$E_z := \begin{pmatrix} 1 & -\check{n}_{zz}\iota & \check{n}_{yz}\iota & \delta_{xz}\iota - \check{n}_{yz}\delta_{zz}\iota^2 \\ \check{n}_{zz}\iota + (\check{n}_{xz}\check{n}_{yz})\iota^2 & 1 & -\check{n}_{xz}\iota & \delta_{yz}\iota - \check{n}_{xz}\delta_{zz}\iota^2 \\ -\check{n}_{yz}\iota + (\check{n}_{xz}\check{n}_{zz})\iota^2 & \check{n}_{xz}\iota + (\check{n}_{yz}\check{n}_{zz})\iota^2 & 1 & \delta_{zz}\iota + \check{n}_{xz}\delta_{yz}\iota^2 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (14)$$

Note that if we set the result corresponds to the classical error matrix.

3.3 Three-axis machine kinematics

We discuss the error kinematics of a three-axis machine by means of arithmetic. The following matrices represent the errors of the tool actual position and orientation.

The first matrix expresses the error of the translation w.r.t. the X axis.

$$R^A_{2\to 1} = \begin{pmatrix} 1 & -\check{n}_{zx}\iota & \check{n}_{yx}\iota & x + a_2 + \delta_{xx}\iota - \check{n}_{yx}\delta_{zx}\iota^2 \\ \check{n}_{zx}\iota + (\check{n}_{xx}\check{n}_{yx})\iota^2 & 1 & -\check{n}_{xx}\iota & b_2 + \delta_{yx}\iota - \check{n}_{xx}\delta_{zx}\iota^2 \\ -\check{n}_{yx}\iota + (\check{n}_{xx}\check{n}_{zx})\iota^2 & \check{n}_{xx}\iota + (\check{n}_{yx}\check{n}_{zx})\iota^2 & 1 & c_2 + \delta_{zx}\iota + \check{n}_{xx}\delta_{yx}\iota^2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (15)$$

where a_2, b_2 and c_2 denotes the offset between the position 2 and 1. Next the error of translation w.r.t. the y axis is of the form

$$R^A_{1\to R} = \begin{pmatrix} 1 & -\check{n}_{zy}\iota & \check{n}_{yy}\iota & a_1 + \delta_{xy}\iota - \check{n}_{yy}\delta_{zy}\iota^2 \\ \check{n}_{zy}\iota + (\check{n}_{xy}\check{n}_{yy})\iota^2 & 1 & -\check{n}_{xy}\iota & y + b_1 + \delta_{yy}\iota - \check{n}_{xy}\delta_{zy}\iota^2 \\ -\check{n}_{yy}\iota + (\check{n}_{xy}\check{n}_{zy})\iota^2 & \check{n}_{xy}\iota + (\check{n}_{yy}\check{n}_{zy})\iota^2 & 1 & c_1 + \delta_{zy}\iota + \check{n}_{xy}\delta_{yy}\iota^2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (16)$$

and finally the translation w.r.t. the z axis.

$$R^A_{3\to R} = \begin{pmatrix} 1 & -\check{n}_{zz}\iota & \check{n}_{yz}\iota & a_3 + \delta_{xz}\iota - \check{n}_{yz}\delta_{zz}\iota^2 \\ \check{n}_{zz}\iota + (\check{n}_{xz}\check{n}_{yz})\iota^2 & 1 & -\check{n}_{xz}\iota & b_3 + \delta_{yz}\iota - \check{n}_{xz}\delta_{zz}\iota^2 \\ -\check{n}_{yz}\iota + (\check{n}_{xz}\check{n}_{zz})\iota^2 & \check{n}_{xz}\iota + (\check{n}_{yz}\check{n}_{zz})\iota^2 & 1 & c_3 + z + \delta_{zz}\iota + \check{n}_{xz}\delta_{yz}\iota^2 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (17)$$

4 CONCLUSION

We proposed an algorithm on the volumetric deviation calculation, see the resulting matrices E_x, E_y and E_z representing the translation errors. We stress that the innovation lies in the automated calculations within the matrix algebra. More precisely, we use the matrix multiplication only, while classically after the evaluation some identities have to be applied to neglect the higher order error terms. This is allowed by the dual numbers matrix representation which, consequently, leads to the kinematic chain assembled from 12×12 matrices. We applied the algorithm on the data obtained by the LaserTRACER measurement of the three-axis machine (MCV 754 QUICK) and compared it to the results calculated classically. The results are the following:

$E_x = -9,4, E_y = 12$ and $E_z = -11$

with classical homogeneous transformation matrices and

$E_x = -9,4, E_y = 14$ and $E_z = -8$

by means of the transformation matrices over the dual numbers. The difference lies in the way how the squarness errors are treated. In our approach, the translational term $\delta_{zz} + \check{n}_{xz}(A0Zz + b_3)$ appears while classically $\delta_{zz} + A0Zz + b_3$ is obtained. The terms combining the $\check{n}_{xz}$ and $A0Z$ errors are redundant and follow from the kinematic chain composition. Their interpretation and their contribution to the overall accuracy is the topic for further research.

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