IDENTIFICATION OF MATERIAL PARAMETRES BY FEM

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The article describes an identification of material parameters from set of experiments. The experiments were made with hollow cylindrical specimens. The specimens were fabricated from the steel (11375). In this study are used the data obtained from 5 experiments (axis force, torque and their combination). Software ANSYS v.11.0 based on Finite Element Method (FEM) was used as a tool for the identification of the material parameters (inverse algorithm). There were implemented many material models in the software. The article uses Multilinear material model, rate independent and Ramberg-Osgood approximation of constitutive equations (3. material parameters) for the problem solution. The data correlations and data dispersions were used for estimating new values of material parameters. The solutions were found by FEM (for selected material model, 3. material parameters) and were compared with the experimental data.

Keywords
inverse algorithm, FEM, experiments, tension, torsion, multilinear isotropic, material parameters

1. Introduction

Identification of the material parameters from the experiment data is the main part of experimental analysis. Finite Element Method (FEM) for material parameters identification was possible to use thanks to more powerful computers with bigger operation capabilities. The basic procedure so-called inverse algorithm is described e.g. in [Aquino 2006]. To the modification of material parameters can be used a lot of different algorithms e.g. Probability algorithm, Neural Networks, Genetic Algorithms, Gradient methods, see [Rojicek 2007]. The Neural Networks are applied in literature most often (e.g. [Aquino 2006]). In the study was used the Probability algorithm based on correlation between the data and the data dispersion. The experiments were accomplished on the Universal Testing Machine on Department of Mechanics of Materials see [Fusek 2007]. The testing machine was designed for testing multiaxial stress state and can be used for either static tests or low cycle fatigue tests see [Frydrysek 2005]. The hollow cylindrical specimens (see Fig. 1) were used for experiments with combined loads (axial force, torque, inner or outer pressure and their combinations). In the hollow specimen are conspicuous two-axial stress states (third main stresses arelow order) during the experiment. Two-axial stress states can be managed and controlled by computer software. The Universal Testing Machine on Department of Mechanics of Materials is described in detail in e.g. [Fusek 2007], [Fojtik 2005].

2. Experiments

The hollow cylindrical specimens (see Fig. 1) used for experiments was fabricated from the steel (11375).

On Fig. 2 are shown three possibilities of loading the specimen (a/ loaded by the axial force – Tension, Compression, b/ loaded by torsion about the axis – Torsion, c/ loaded by combination of both a/ and b/ – Combin). The specimen can be also loaded by inner or outer pressure; it was not used in this study.

The applications of loads in experiments were controlled by the deformation (linear increased elongation y, twisting angle φ). The loads

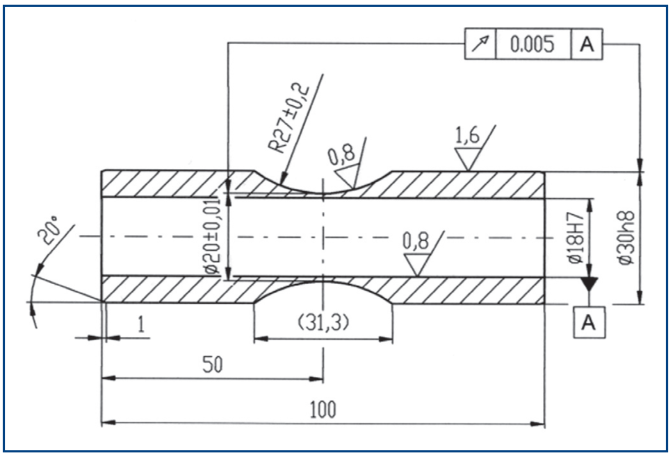

Figure 1. *The Specimen*

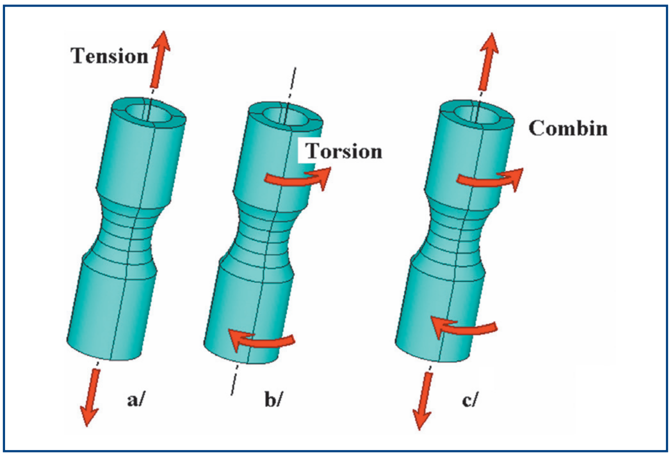

Figure 2. *Loading variants*

in all experiments are managed by elongation value y and twisting angle value φ (in case of combinations the twisting angle values φ were calculated from the elongation value y according equations inside of Tab. 1). At the same time was measured torque value Mk and axial force value F. The loading variation called Tension displayed null torque value Mk (negligible) and thus was excluded from consideration. The loading variant called Torsion displayed low axial force value F (negligible) and thus was ignored for calculation too.

Five experiments were performed:
Tension – axial force, elongation,
Torsion – torque, twisting angle,
Combin_1 – axial tension force and torque,
Combin_2 – axial tension force and torque,
Combin_3 – axial compression force and torque.

The specimen loads are described in Tab.1.

Table 1. Application of loads

Type:	Torque Mk	Axial Force F	Twisting angle φ [rad]	Elongation y_{MAX} [mm]
Tension	Measured	Measured	0	1.67
Torsion	Measured	Measured	φ_{MAX} = 1.484	0
Combin_1	Measured	Measured	y×(5×π/180)/0.25	1.36
Combin_2	Measured	Measured	y×(5×π/180)/0.1	0.86
Combin_3	Measured	Measured	-y×(5×π/180)/0.1	–1.98

3. Algorithm

The basic algorithm is described by Fig.3.

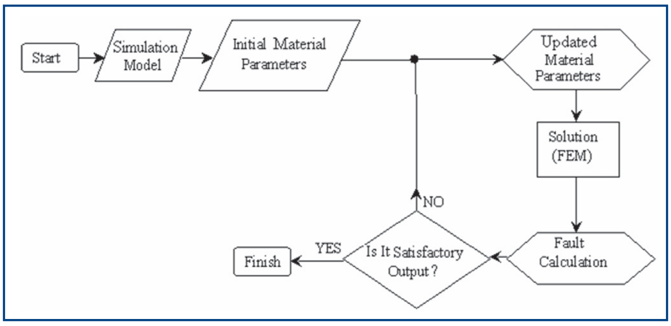

Figure 3. *Basic algorithm*

The first point of algorithm diagram (Fig.3) is building of Simulation Model. Basic design of specimen is included in the simulation model – geometric model (see Fig.1, Fig. 2), finite element model and boundary conditions (loads and deformations) (see Fig. 4). The basics of boundary conditions are experimental data (y, φ see Tab.1) applied to pivot A, the measured value of loads corresponding to reaction loads from FEM (pivot A). The simulation model is shown on Fig.4. The simulation model does not include assigned material parameters (which are inserted later). Element type was selected with respect to appearance of buckling and large deformation effects.

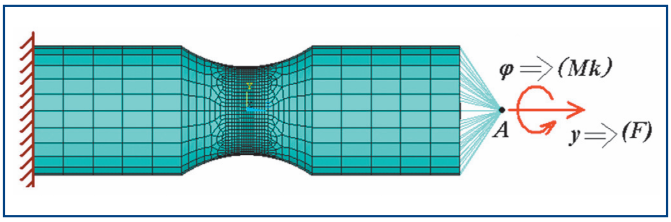

Figure 4. *The simulation model*

The first step of calculation starts with the initial parameters (material). The initial parameters were calculated from first experiment (Tension). Measured value of axial force F was recalculated to stress σ. The measured value of elongation y was recalculated to strain ε. The results (set of points σ, ε) are smoothed by Ramberg-Osgood approximation of constitutive equation (1) in terms of probability algorithm (in more detail see [Rojicek 2007]).

$$\varepsilon = \frac{\sigma}{C1} + \left(\frac{\sigma}{C2}\right)^{\frac{1}{C3}} \tag{1}$$

Final values the initial estimates of material parameters (C1, C2, C3) are shown in Tab. 2. The initial estimates of material parameters have effect to number of calculation cycles. Interesting solution hollow cylindrical specimen is described in [Fuxa 2000].

Table. 2 Initial values of material parameters			
	C1 [MPa]	C2 [MPa]	C3 [1]
Initial parameters (Tension)	196000	759	0.15

The next step of the algorithm is updating of material parameters. The method used for updating – modification material parameters is main part of solution. In this study was used probability algorithm – based on the data correlation and the data dispersion. In first section was calculated a number (e.g. 10) of FE solutions with randomly generated parameters (C1, C2, C3). These solutions (material parameters) are set of input data. The set of output data contains relevant error of solution (The errors calculation is described in next paragraph).The correlation between input and output data and data dispersion can be calculated by equations known from the Statistics (2).

$$\mu_r = E(X^r) = \sum_k x_k^{\;r} \; p(x_k),$$
$$\mu_r^\circ = E((X-\mu)^r) = \sum_k (x_k-\mu)^r \; p(x_k),$$
$$D^2(X) = \mu_2^\circ = \mu_2 - \mu^2, \tag{2}$$
$$K_{xy} = E(XY) - E(X)E(Y),$$
$$\rho_{xy} = \frac{K_{xy}}{D(X)D(Y)}.$$

Where μ_r is r-th general moment random variable *X*, $p(x_k)$ is probability density function random variable *X*, μ_r° is r-th central moment random variable *X*, *E(X)* is mean value random variable *X*, $D^2(X)$ is dispersion random variable *X*, K_{xy} is covariance random variables *X*, *Y* and ρ_{xy} is correlation random variables *X*, *Y*. The new value of parameters e.g. C1 consists of three parts (3):

$$C1 = C10 + random\_1 + corel\_1. \tag{3}$$

– Value of parameter C10 corresponds to solution with minimum Error:

Given $\mathbf{a}_i = (Error_i, C1_i, C2_i, \ldots Ck_i)$ and $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots \mathbf{a}_m\}$

where $\forall \mathbf{a}_i \in \mathbf{A} : Error_i \le Error_{i+1};\; i \in \langle 1, m-1 \rangle$ then $C10 = C1_1$.

k is number of parameters, m is number of previous solutions.

– Value of component random_1 (uniform density function):

$$p(x) = \frac{1}{Max}; \forall x \in (0, Max); \; Max \approx 100 - 10000;\; Step \approx 10 - 1000$$

then $$random\_1 = C1_1 \cdot \left(1 + \left(x - \frac{Max}{2}\right) / (Max \cdot Step)\right).$$

– Value of movable component corel_1 (correlation between C1 – Error, dispersion C1):

Given $del_i = (Error_i - Error_{i+1}, C1_i - C1_{i+1}, \ldots Ck_i - Ck_{i+1}) = (del1_i, del2_i, \ldots delk_i)$

where $i \in \langle 1, m-1 \rangle$.

Calculated: $E(Error) = \sum_{l=1}^{m} Error_l \frac{1}{m}$; $E(C1) = \sum_{l=1}^{m} C1_l \frac{1}{m}$; $E(C2)$; … $E(Ck)$.

Likewise, in accordance with (2)

- $D(Error), D(C1), D(C2), \ldots D(Ck)$.
- $E(del1), E(del2), E(del3), \ldots E(delk)$.
- $D(del1), D(del2), D(del3), \ldots D(delk)$.
- $k_{ERR\_C1} = E(Error \cdot C1) - E(Error) \cdot E(C1)$;
- $k_{ERR\_C2} = E(Error \cdot C2) - E(Error) \cdot E(C2)$;
- …
- $\rho_{ERR\_C1} = \frac{k_{ERR\_C}1}{D(Error) \cdot D(C1)}$;
- $\rho_{ERR\_C2} = \frac{k_{ERR\_C}2}{D(Error) \cdot D(C2)}$;
- …
- $del\_k_{ERR\_C1} = E(del1 \cdot del2) - E(del1) \cdot E(del2)$;
- $del\_k_{ERR\_C2} = E(del1 \cdot del3) - E(del1) \cdot E(del3)$;
- …
- $del\_\rho_{ERR\_C1} = \frac{del\_k_{ERR\_C}1}{D(del1) \cdot D(del2)}$;

- $del\_\rho_{ERR\_C2} = \dfrac{del\_k_{ERR\_C2}}{D(del1)\cdot D(del3)}$;

- ...

Then $corel\_1 = \rho_{ERR\_C1}\cdot D(C1)\cdot\left(1+del\_\rho_{ERR\_C1}\right)/koef$, $koef \approx 2-10$.

For larger number of parameters can be used partial correlation coefficient etc. Detailed information can be found in e.g. [Skrasek 1990].

The static analyses was made in software ANSYS. The MISO (Multilinear isotropics) material model (rate independent) was used for the calculations. The calculations respect buckling and large deformation effects.

The fault of actual solution step was solved by compare values of forces, moments get from FE solution and values of forces, moments get from experiments. The principles of solution are shown on Fig. 5, the Error value [%] was calculated by equation (3).

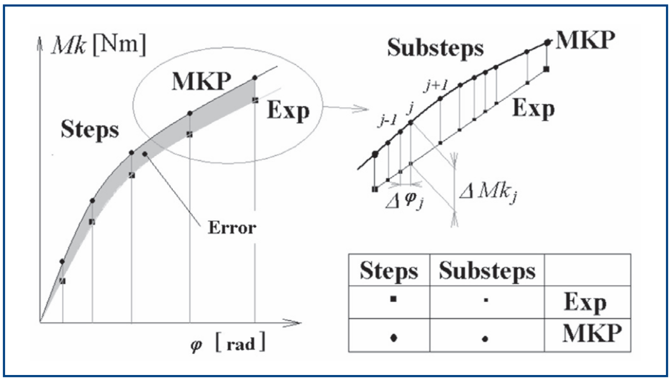

Figure 5. *Error analyze*

$$ERROR = \frac{\sum_j \left\{\left(Mk_j^{Exp} - Mk_j^{FEM}\right)\cdot\left(\varphi_j - \varphi_{j-1}\right)\right\} + \sum_k \left\{\left(F_k^{Exp} - F_k^{FEM}\right)\cdot\left(y_k - y_{k-1}\right)\right\}}{\sum_j \left\{Mk_j^{Exp}\cdot\left(\varphi_j - \varphi_{j-1}\right)\right\} + \sum_k \left\{F_k^{Exp}\cdot\left(y_k - y_{k-1}\right)\right\}}\cdot 100 \qquad (3)$$

The algorithm was terminated after prescribed value of ERROR or prescribed value of number of solution cycles was obtained.

4. Results

For calculation was used computer with: processor Core 2 Duo E6420, 2x1024MB DDR2 800 RAM, 320GB SATAII/300 7200RPM (software ANSYS v.11.0 – MKP, Borland DELPHI for Windows 2005, 2007 – probability algorithm, value analyses etc.). Fifty computational cycles were performed for the solution of each experiment (finding of parameters). One computational cycle took about 10-15 min. – the one experiment solving ran about 12 hours. The probability algorithm gives different results in each simulation.

The material parameters (C1, C2, C3 see equation (1)), with the errors found by algorithm, are displayed on Tab. 3.

Table. 3 Material parameters

	C1 [MPa]	C2 [MPa]	C3 [1]	Error [%]
Tension	199428	759	0.2009	1.26
Combin_1	193501	713	0.2005	4.70
Combin_2	195653	681	0.2162	4.26
Torsion	194241	662	0.2044	6.52
Combin_3	190482	603	0.2075	5.75

Comparing of the FE results and experiments are shown on Fig. 6.

5. Data interpretation

From the comparing of the results (see Tab. 3, Fig. 7) are evident interesting outcomes. The decreasing of the axial stresses in the specimen is caused the decreasing of the parameter C2 (curve move

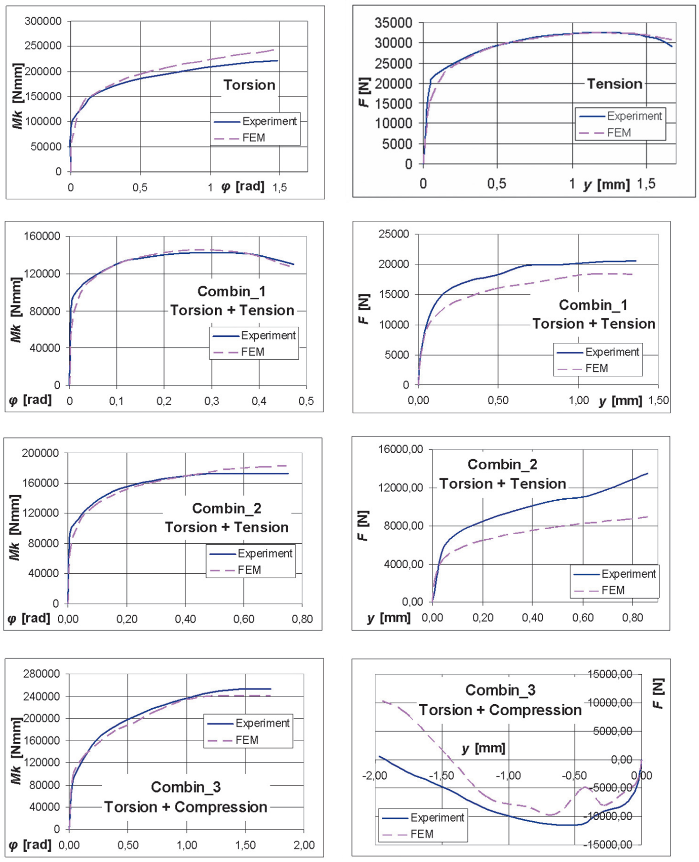

Figure 6. *Compare results FEM analyses and experiments*

to the right). It is evident that the selected material model (multilinear isotropic) is not able to correctly describe the behavior of tested material.

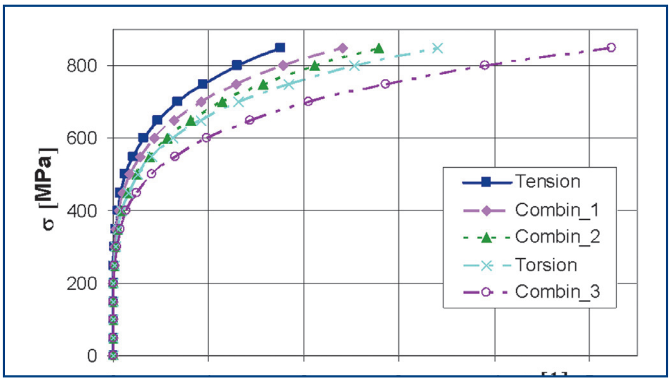

Figure 7. *The results*

6. Conclusions

In this article is described identification of material parameters (3 parameters) in conditions of isotropic material (ANSYS – MISO material model). Finite Element Method (FEM) – software ANSYS v.11.0 was used for identification of the material parameters by the inverse algorithm. The material parameters were identified in 50 cycles from tested experiments. Maximum Error – difference between solution by FEM and experiments was less then 7%. The material parameters, which were determined by the inverse method, are closer to the reality.

The results above indicate that the material shows different behavior for different types of loading. The correlation between material parameters (C2) and loads were determined.

The inverse method proposed in this study is easily applicable to other material models for static or low cycle fatigue tests. The next step of the analysis will be the identification of parametres for anisotropic or kinematics material models (e.g. ANSYS-Hill anisotropy + MISO, Chaboche).

Acknowledgements
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