MULTIPHASE FLUID MODELS TO DEAL WITH FLUID FLOW DYNAMICS

MILADA KOZUBKOVA[1], JANA JABLONSKA[1], MARIAN BOJKO[1], LUKAS DVORAK[1], MARIA CARNOGURSKA[2]

[1]Departament of Hydromechanics and Hydraulic Equipment, VSB Technical University of Ostrava, Ostrava, Czech Republic

[2]Institute of Structural and Process Engineering, Technical University of Kosice, Kosice, Slovakia

DOI : 10.17973/MMSJ.2019_06_201880

milada.kozubkova@vsb.cz

When dealing with dynamic issues, we often encounter problems of hydraulic shock (water hammer) and cavitation causing distortion of the surrounding material, destruction of material, accompanied by sounds and vibrations. These dynamic behavior of the liquids is due to the presence of gases in the liquid, especially air, vapor and possibly other gases in smaller quantities. The density of such a liquid is assumed to be a function of a liquid elastic modulus depending on pressure or it is defined as the density of a multiphase mixture of incompressible liquid and compressible gases (vapour, air) depending on pressure too. The article is focused on specification of mathematical models of multiphase flow for piping (one dimensional) hydraulic systems and spatial (three dimensional) hydraulic elements and systems. The electrohydraulic analogue (Matlab-SimHydraulics) method and finite volume method (Ansys-Fluent) are used for illustrative fluid dynamics tasks.

KEYWORDS

multiphase fluid models, modulus of elasticity, density of compressible liquid, fluid dynamics, CFD

1 INTRODUCTION

Hydraulic liquids include gas which is the main cause of the dynamic behavior of hydraulic systems. Due to the presence of gas in the liquid, its compressibility is defined by non-constant pressure-dependent density in two ways:
- density of multiphase mixture of liquid and gases
- density defined by the modulus of elasticity

The density of the mixture is a function of the individual phase density, each density can be pressure dependent. The modulus of elasticity is usually determined experimentally, while not considering its dependence on time. Both definitions depend on the gas content in liquid, which can be determined experimentally by static measurements. While mixture flowing, the pressure of the system is changing, therefore gas volume changes too. Its value is then corrected experimentally. Accurate specification of all physical properties of fluids is a necessary prerequisite for creating a suitable mathematical model of static and dynamic fluid flow. When dealing with the dynamics of hydraulic systems we often encounter the issue of cavitation. Cavitation depends on the type of liquid used in hydraulic system and multiphase mixture consisting of liquid, air and vapor [Sikora 2015].

The simulation examines the dynamic properties of the system to determining the time dependence of the required quantities. For simulation, a mathematical model is comprised of algebraic, partial differential and ordinary differential equations. Modeling is done analytically (exactly, Laplace's transformation for linearized cases) or numerically (Euler method, Runge-Kutta method, finite volume method, etc.) [Kozubkova 2009].

Hydraulic circuits consist of hydraulic elements and a piping system. When dealing with the construction of hydraulic elements, it is necessary to solve the complete multidimensional system of momentum and continuity equation. The result is the distribution of pressure and velocities, respectively flow rates in the whole area. However, the boundary conditions that affect the solution must be taken into account [Kozubkova 2016], [Čarnogurská 2015].

Conventional hydraulic circuits often connected by long pipelines are unresolvable in the above described manner due to the time requirements. Therefore simpler models of hydraulic pipes, elements and circuits, which in their topology resemble electrical circuits, are derived. Also, the individual elements exhibit formally analogous properties including the definition of hydraulic resistances against motion [Miller 1990], hydraulic inertia and hydraulic capacities. The liquid column then corresponds to the power line, including the long lead line. Thus, the hydraulic element variables are assumed to be a function of time and are independent of the spatial coordinate in case of one-dimensional model. The liquid column flow can be a function of time and, optionally, one spatial coordinate. Therefore, solution of simplified system of equations in analogies with electrical circuit is approached. This simplification brings a number of problems with definition of liquid compressibility, friction, etc. However, this method is processed into a number of commercial programs. The highest quality program is Flowmaster [Kozubkova 2009], now extension of Matlab Simulink has been developed, i.e. SimHydraulics [Kozubkova 2016].

The above-described system of equations applied to the hydraulic circuit is solvable in two ways:
- spatial solution of the whole system of equations by finite volume method,
- solution by a simplified approach called electrohydraulic analogy, resulting from certain similarities of hydraulic and electrical circuits

2 MATHEMATICAL MODEL OF COMPRESSIBLE LIQUID

Compressible liquid is a specific by functional dependence of density to pressure, while the main problem here is the amount of air in a liquid. Basic air content information in liquid gives Henry's theory [Brennen 1995], it is also possible to experimentally determine this quantity (usually by measuring the amount of oxygen in the water and from given ratio of oxygen and nitrogen in the absence of other gases, the amount of air in molar, volume or mass fractions can be determined). The liquid´s modulus of elasticity is a significant parameter used in technical practice. Its value may be constant or may be a function of pressure and volume fraction of air. Then also the density will not be constant.

2.1 Solubility

The amount of gas that can be absorbed by water is described by the following equation [Himr 2010]:

$$p_i = x_i \cdot H \tag{1}$$

Molar fraction x_i here refers to the amount of dissolved gas component in water. H expresses Henry's constant. Partial gas pressure above the surface p_i can be determined from formula:

$$p_i = x_i \cdot p \tag{2}$$

where x_i denotes the molar fraction of the components in gaseous mixture above the surface and p is the total pressure above the surface. The definition of molar fraction is:

$$x_i = \frac{n_i}{\sum_i n_i} \tag{3}$$

where n_i denotes the amount of gas substance and $\Sigma_i n_i$ denotes the amount of mixture.
Substance amount can be calculated according to:

$$n = \frac{m}{M} \tag{4}$$

where m is the mass and M is molar mass.
Conversion between molar and mass fraction is followed:

$$w_i = \frac{x_i M_i}{\sum_i x_i M_i} \tag{5}$$

In our case, mixture of gases in the air is composed of Oxygen (O_2) and Nitrogen (N_2) with exclusion of other substances, which are contained in the air in trace amounts. Mass fraction of O_2 is w_{iO2}=0.23 [-], while mass fraction of N_2 is w_{iN2}=0.77 [-], which together gives mass fraction of air w_{air}=1 [-]. The resulting mixture(Air) is of Henry´s constant H=6,6135.10^9 [Pa]. All calculations were done for temperature T=293,15 [K] and absolut pressure p=10^5 [Pa].

2.2 Multiphase flow – model of liquid and gas mixture

The spatial model of the mixture is a simplified multiphase model that can be used for multiphase flow, where the phases move at different velocities, but the balance is assumed at a short spatial scale, [Shaughnessy, Jr. 2005] ,[Kozubkova 2009]. The bond between the phases is very strong. This model can also be used to solve a homogeneous multiphase flow with a very strong bond between phases moving at the same velocity. The model can solve the flow of n-phase of liquid or particles by solutions of momentum equation and continuity equation for the mixture, the equations for the volume fraction of other phases and algebraic relationship for the relative velocity.

The continuity equation for the mixture is given by the relation

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial(\rho_m u_{m,j})}{\partial x_j} = 0 \tag{6}$$

where $u_{m,j}$ are the velocity components averaged by weight, (index j is Einstein summation index).

$$u_{m,j} = \frac{\sum_{k=1}^{n} \alpha_k \rho_k u_{k,j}}{\rho_m} \tag{7}$$

and

$$\rho_m = \sum_{k=1}^{n} \alpha_k \rho_k \tag{8}$$

is the density of the mixture and α_k is volume fraction of the phase k. If some phase is gaseous, gas density is calculated from the state equation

$$\rho_g = \frac{p_{ref} + p}{rT_{ref}} \tag{9}$$

where r is the specific gas constant, T_{ref} is the static temperature. In case of a compressible medium, the volume fraction of the phase is influenced by the pressure [1].

The momentum equation for the mixture is obtained by summing the momentum equations for the individual phases

$$\begin{aligned} &\frac{\partial(\rho_m u_{m,i})}{\partial t} + \frac{\partial}{\partial x_j}(\rho_m u_{m,i} u_{m,j}) = \\ &-\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu_m\left(\frac{\partial u_{m,i}}{\partial x_j} + \frac{\partial u_{m,j}}{\partial x_i}\right) - \mu\delta_{ij}\frac{2}{3}\frac{\partial u_{m,l}}{\partial x_l}\right) + \\ &+\rho f_i + \frac{\partial}{\partial x_j}\left(\sum_{k=1}^{n} \alpha_k \rho_k u_{dr,km,i} u_{dr,,k,j}\right) \end{aligned} \tag{10}$$

where n is the number of phases, f_i are components of external mass forces.

$$\mu_m = \sum_{k=1}^{n} \alpha_k \mu_k \tag{11}$$

is the dynamic viscosity of the mixture. The specification of the drift velocity depends on the definition of the resistive forces of the particles.
The secondary fraction volume equation is

$$\begin{aligned} &\frac{\partial(\alpha_p \rho_p)}{\partial t} + \frac{\partial(\alpha_p \rho_p u_{m,j})}{\partial x_j} = -\frac{\partial(\alpha_p \rho_p u_{dr,p,j})}{\partial x_j} \\ &+ \sum_{q=1}^{n}(Q_{m,qp} - Q_{m,pq}) \end{aligned} \tag{12}$$

The slip velocity is defined as difference of the phase velocity p and phase velocity q $u_{p,q,l} = u_{p,i} - u_{q,i}$.

In cavitation, the gaseous phase consists of air and vapor, and therefore it is necessary to add an equation addressing the dynamics of the vapor bubbles. Additionally, the model can handle flow of non-Newtonian liquids. The three-dimensional model can be simplified, to address piping systems, to one-dimensional single-phase model, where the density is a function of the modulus of elasticity.

2.3 Single-phase flow – model of compressible liquid

Basic equation of nonstationary flow of compressible liquid in pipeline (one - dimensional task) is also derived from continuity and Navier - Stokes equations, assuming that the change in velocity according to the coordinate in the pipeline is very small against the change in velocity by time. Continuity equation along with the equation of momentum forms the system of partial differential equations of the first order of the hyperbolic type:

$$\frac{\partial u}{\partial x} + \frac{1}{K}\frac{\partial p}{\partial t} = 0 \tag{13}$$

$$\frac{\partial u}{\partial t} + \frac{2\lambda}{d}u|u| + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

Viscous member expressed the effect of friction by means of a friction coefficient λ. For pipelines, further simplification is possible, where the derivation according to the spatial variable is expressed by the differential fraction in relation to the length of the pipe *l*, i.e.

$$\frac{\partial p}{\partial x} = \frac{\Delta p}{l} \text{ and } \frac{\partial Q}{\partial x} = \frac{(Q - Q_0)}{l}.$$

Then the system of partial differential equations passes over to a system of ordinary differential equations:

$$\frac{dp}{dt} = -\frac{1}{C}Q \text{ for } Q_0 = 0 \tag{14}$$

$$\Delta p + RQ|Q| + L\frac{dQ}{dt} = 0$$

where resistance against the acceleration *L* and the inverse resistance value against deformation *C*, i.e. the capacity resulting from electro-hydraulic analogies, are defined for the entire length of the pipeline and the resistance to movement *R* is distinguished for laminar and turbulent flow, see liter.: The hydraulic capacity depends on the modulus of elasticity, whereas the resistance against acceleration only on the physical and geometric parameters. Resistance to movement R is a function of viscosity [Fajka 2017].

2.4 Density and modulus of elasticity of compressible liquid

Density of the multiphase mixture of liquid and gas ρ_{mix} is given by the basic relationship:

$$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g \tag{15}$$

where α_l is liquid volume fraction, ρ_l is density of liquid, α_g is gas volume fraction and ρ_g is the gas density at a given reference pressure and reference temperature [Bird 2002].

Density of single-phase compressible liquid ρ connects with module of elasticity. The compressibility of the fluid is the property of reducing its volume when increasing the external pressure relative to reference value (e.g., measured value). Density is than expressed in dependency on modulus of elasticity *K*, respectively compressibility factor δ = *1/K* [Jablonska 2014], [Kopacek 1986], [Kozubkova 2009].

$$d\rho = \frac{\rho}{K}dp \tag{16}$$

It means

$$\rho = \rho_{ref}\left(1 + \frac{(p - p_{ref})}{K}\right) \tag{17}$$

The modulus of elasticity may be constant or significantly dependent on pressure and volume fraction of gas in liquid. From the state equation for polytropic change the volume of gas depending at relative pressure can be derived:

$$p_{ref}V_{g,ref}^n = (p_{ref} + p)V_g^n \rightarrow V_g = V_{g,ref}\left(\frac{p_{ref}}{p_{ref} + p}\right)^{\frac{1}{n}}$$

n is a polytropic coefficient and then a volume fraction of gas at relative pressure *p* is

$$\alpha_{gp} = \alpha_{g,ref}\left(\frac{p_{ref}}{p_{ref} + p}\right)^{\frac{1}{n}} \tag{18}$$

Var.	Model	Density/Modulus of elasticity
A_{mix}	multi phase flow	$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g$
B_{exp}	single phase flow	$\rho_{exp} = \rho_{ref}\left(1 + \frac{(p - p_{ref})}{K_{exp}}\right)$ K_{exp}=const.
B_{th}	single phase flow	$\rho_{th} = \rho_{ref}\left(1 + \frac{(p - p_{ref})}{K_{th}}\right)$ $K_{th.}$=const.
B_{hyd}	single phase flow	$\rho_{hyd} = \rho_{ref}\left(1 + \frac{(p - p_{ref})}{K_{hyd}}\right)$ $K_{hyd} = \frac{K_{th}n(p_{ref} + p)}{n(p_{ref} + p) + \alpha_{g,ref}(K_{th} - n(p_{ref} + p))}$
B_{matlab}	single phase flow	$\rho_{matlab} = \rho_{ref}\left(1 + \frac{(p - p_{ref})}{K_{matlab}}\right)$ $K_{matlab} = K_{th}\frac{1 + \alpha\left(\frac{p_{ref}}{p_{ref} + p}\right)^{1/n}}{1 + \alpha K_{th}\frac{p_{ref}^{1/n}}{n(p_{ref} + p)^{\frac{n+1}{n}}}}$

Table 1. Density definitions for various modulus of elasticity

In Tab. 1 is a summary of density definitions for various mathematical models. For water, it may contain air according to Henry's relationship at given pressure and temperature reference values.

3. ESTIMATION OF MODULUS OF ELASTICITY AND DENSITY WITH AIR IN WATER

The above relationships for the modulus of elasticity and density were subsequently verified on a hydraulic hammer case [Fajka 2017]. The hydraulic circuit consisted of a tank, a pump (H=4 m) and a long pipe of 0.025 m in diameter and a length of 48 m. After flow stabilization, the circuit was stepped closed by a ball valve (closing time was 0.1 s). A static pressure measurement was performed at the beginning and end of the long pipe, see Fig. 1, from which it was possible to determine the period of dynamic action and then the experimental modulus of elasticity, taking into account the air content of fluidity and elasticity of the wall. [Shaughnessy, Jr. 2005], [Burecek 2015].

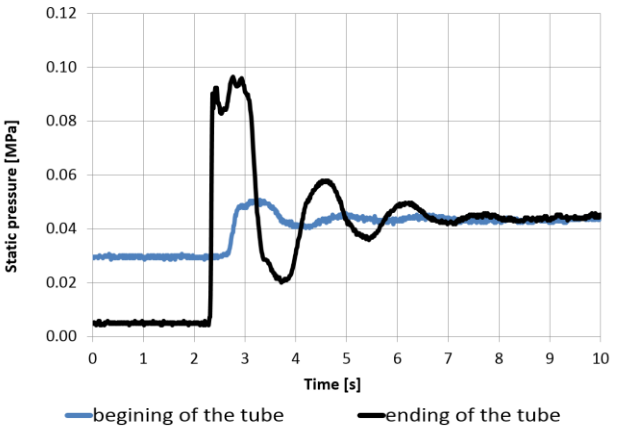

Figure 1. Static pressure at the beginning and the end of the pipe during hydraulic shock

For comparison on Fig. 2 the dependence of elasticity modulus of water versus the pressure calculated for different model variants is plotted [Fajka 2017]. The theoretical modulus of elasticity K_{th} equals 2100 MPa and the experimentally determined modulus of elasticity K_{exp} equals 12,4 MPa (using period given in Fig. 1).

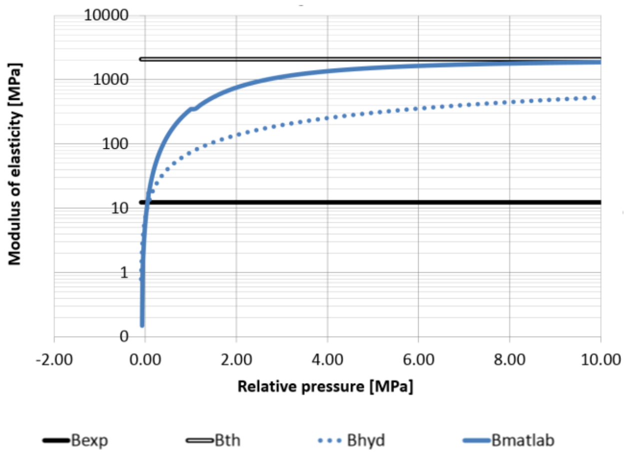

Figure 2. Water module of elasticity vs. pressure

At reference conditions (20 ° C, 0,1 MPa) the density of water is 998.2 kg/m3 and the volume fraction of air is 0.02%. It can be seen from Fig. 2, that the theoretical modulus of elasticity (variant B_{th}) significantly exceeds other variants, in particular at the lower pressures, which results unrealistic results in modeling of dynamic tasks and causes divergence in numerical calculations, see Fig. 3. Other variants get closer.

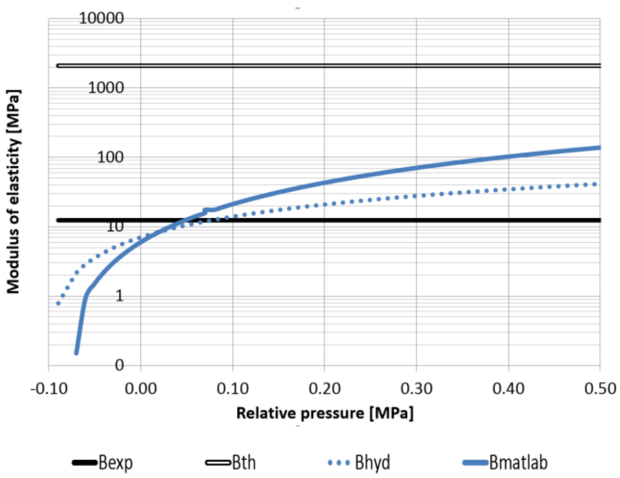

Figure 3. Water module of elasticity vs. pressure (detail)

Approximately the order of the density using modulus of elasticity determined experimentally ([Kozubkova 2016]) and the density models used in dynamic tasks (e.g. water hammer) appear to be real, see Fig. 4 and 5. It is essential that the densities determined by all model variants match the reference value measured at the reference conditions. For variant A_{mix} it means that the reference density is the density of the mixture.

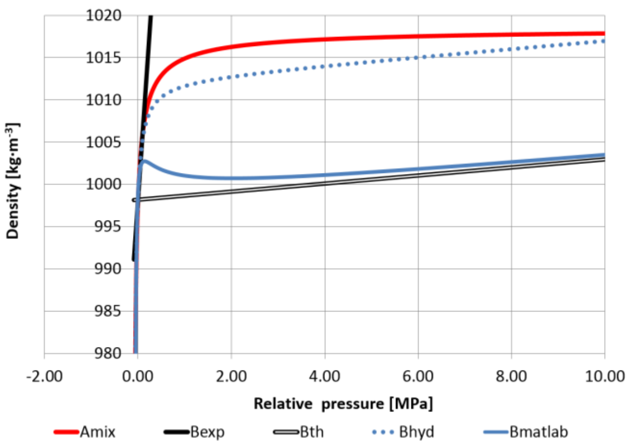

Figure 4. Water density vs. pressure

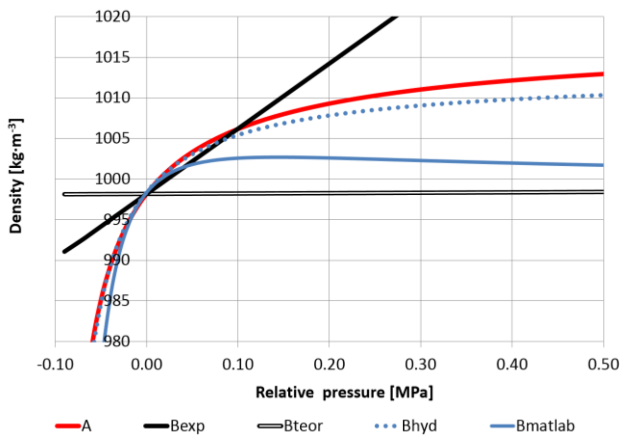

Figure 5. Water density vs. pressure (detail)

It is possible to compare the modulus of elasticity as a function of the volume fraction in the range of pressures corresponding to the water hammer case. In Fig. 6 the curve of the experimental modulus of elasticity K_{exp} intersects the curve of the modulus of elasticity K_{matlab} (2% volume fraction of air, K_{th} = 2 100 MPa) at relative pressure 0,04 MPa approximately.

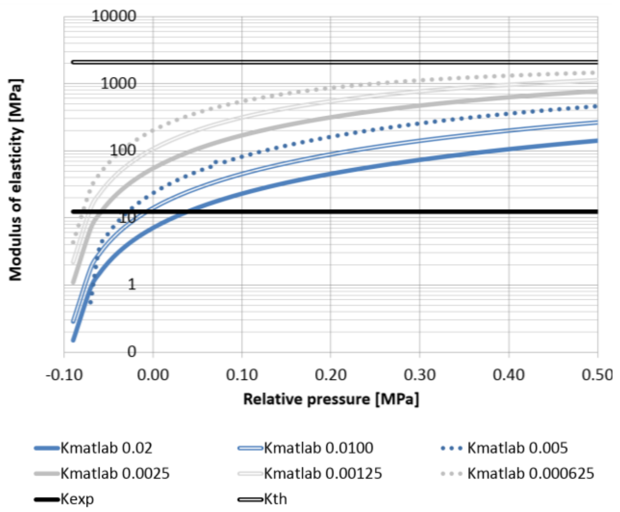

Figure 6. Water´s modulus of elasticity vs. pressure and volume fraction of air

This value of pressure was measured at steady state of experiment (closed valve) and modulus of elasticity K_{exp} or K_{matlab} (2% air volume fraction, K_{th} = 2 100 MPa) is optimal value for the numerical modeling of both electrohydraulic analogy models (B_{exp} and B_{matlab}) and multiphase model (A_{mix}). For low experimental pressure values, density values are slightly different for A_{mix} (mixture of water and 2% volume fraction of air) and B_{matlab} (2% volume fraction of air, K_{th} = 2 100 MPa) variants, see Fig. 7. Density for small volume fraction of air is nearly constant (0.005, 0.0025, 0.00125% volume fraction of air)

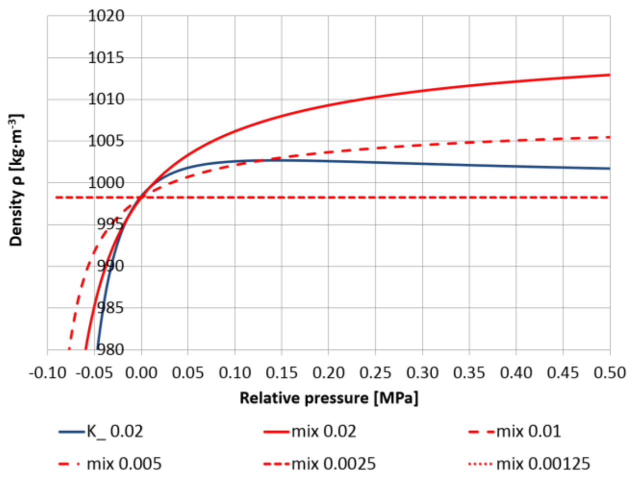

Figure 7. Water density vs. pressure and volume fraction of air

4. RESULTS OF MATHEMATICAL MODELING OF WATER HAMMER

The water hammer was modeled by all model variants (see Table 1) and the results compared with the experiment.

The single-phase model was solved using Matlab software. The boundary condition at the beginning of the pipe was defined by the measured pump characteristic. At the end of the pipe, the valve closure was specified as time dependence. The solution of the system of ordinary differential equations converged rapidly thanks to a variable time step.

For the multiphase model, the boundary condition at the input was defined in the same way; at the outlet the valve closure was determined by the time change of the flow to zero. The time step was selected to be 0.001 s, which conformed to the convergence conditions. ANSYS Fluent software for axisymmetric task of flow in pipeline was used.

The quality of the selected model varioants is evident from graphs of comparison of calculation and measurement, see Fig. 8 and Fig. 9.

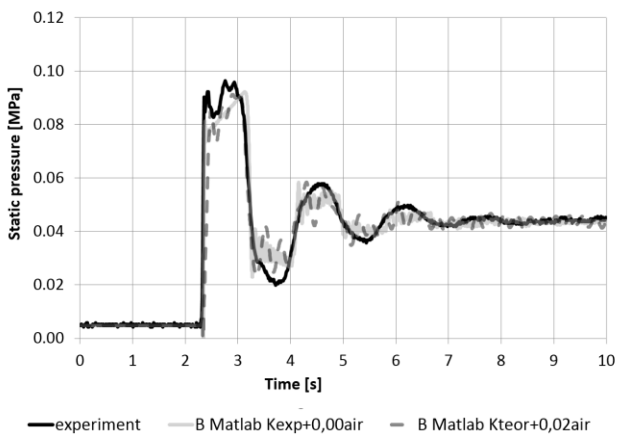

Figure 8. Results of static pressure in dependence on time gained by experiment and simulation by Matlab (B_{exp}, B_{th})

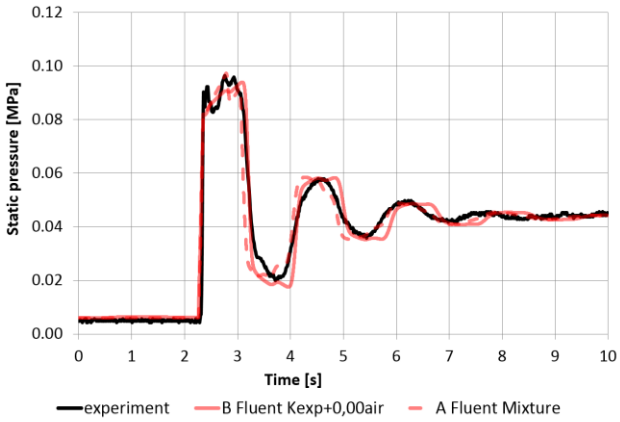

Figure 9. Results of static pressure in dependence on time gained by experiment and simulation by Fluent (Bexp, A)

5 CONCLUSION

The accuracy of the numerical solution of hydraulic system dynamics strongly depends on the physical properties of the fluid and the accuracy of the hydraulic element static characteristics [Miller 1990]. Physical properties are influenced by measured oxygen content and consequently by calculated air content in used liquid. There are several mathematical relationships for definition of the modulus of elasticity and density. Mathematical models of fluid flow differ in number of phases or in dimensions (one dimensional method (SimHydraulics) and three dimensional methods (ANSYS – Fluent)). On Fig. 8 and Fig. 9 the comparison of tested approaches is shown. The methods are well comparable and differ in terms of simplicity, respectively complexity of geometry and calculation time. It has been confirmed, that for dynamic hydraulic tasks it is necessary to consider the liquid as compressible liquid or as a mixture of liquid and air. This assumption will also be used to examine cavitation where the flowing medium will be considered as a multiphase mixture of water, air and water vapor. The liquid will be considered as incompressible liquid and water vapor and air as a compressible gas.

ACKNOWLEDGMENTS

This work was supported by the European Regional Development Fund in the Research Centre of Advanced Mechatronic Systems project, project number CZ.02.1.01/0.0/0.0/16_019/0000867 within the Operational Programme Research, Development and Education.

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

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The work presented in this paper was supported by a grant SGS „Modeling and Experimental Verification of Dynamic Phenomena in Fluid and Vacuum Systems" SP2018/57.

CONTACTS:

Prof. RNDr. Milada Kozubkova, CSc.
VSB Technical University of Ostrava, Departament of Hydromechanics and Hydraulic Equipment,
17. listopadu 15/2172, CZ- 70830 Ostrava
+420 597 323 342, milada.kozubkova@vsb.cz

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