THE PROCESS OF CREATING A COMPUTATIONAL 3D MODEL OF A HARMONIC TRANSMISSION

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The article focuses on the explanation of the basic principles and functions of harmonic gear systems. The main objective of the article is to comprehensively explain the relations concerning the main construction parameters of these special gear systems and their impact on the backlash. It also includes basic mathematical relationships between the reaction elements, mechanical characteristics, cogging types and the properties of the harmonic gear systems. This article focuses mainly on the preparation of the finite element 3D computational model of the harmonic drive gear and tooth backlash analysis. The results of the simulation were analysed in the final part of the article.

KEYWORDS:
Harmonic gear systems, construction parameters, 3D computational model, backlash analysis

1. INTRODUCTION TO THE TOPIC OF HARMONIC GEAR SYSTEMS

In the research we focus on the description of the construction of the geometry of harmonic gear systems and the construction of the 3D model for a numerical simulation. In the introduction, the article deals with the issue of harmonic gear systems. In the next chapter the process of construction and design of individual elements of a harmonic gear system is described. Furthermore, an issue of a preparation of the 3D numerical model is provided including an analysis of the generation of the numerical 3D network as well as the preparation of a numerical model. In the following chapter the process of the design of the 3D model of the harmonic gear system is presented. In conclusion, the results of measurements and summary of findings are presented.

In 1955, American inventor, Clarence Walton Musser, issued a new gear concept called "Strain Wave Gearing"; later, in 1957, the mechanism under this name was patented as "Harmonic Drive Gearing, Strain Wave Gearing", see in Figure 1. In 1959, for the first time, this concept raised many questions concerning the revolutionary solution of a mechanical gear with a flexible spline. This concept was in opposition to the conventional principles of rigid body dynamics applied in conventional gearing systems. The invention of W. Musser was based primarily on the dynamics of an elastic body that remarkably differentiated it from the former acknowledged idea of rigid gear systems. The first prototypes of harmonic drive gearing systems were made in 1958 in Massachusetts, America, by United Machinery Group Corp. Harmonic Drive Division managed the production. Currently, three biggest global companies are producing and developing the harmonic drive gearing systems, namely: Harmonic Drive LLC (USA), Harmonic Drive Systems (Japan), and Harmonic Drive AG (Germany). Harmonic drive gearing systems are widely used in the area of precise control. A precise reduction and positioning are mainly used in robotics. Its high rate of precision and weight is applied in the aviation industry. The system is also used in the engineering industry for transmission of high torque, conveyor and turbine drives, and in many other machine applications [Brumercik 2017].
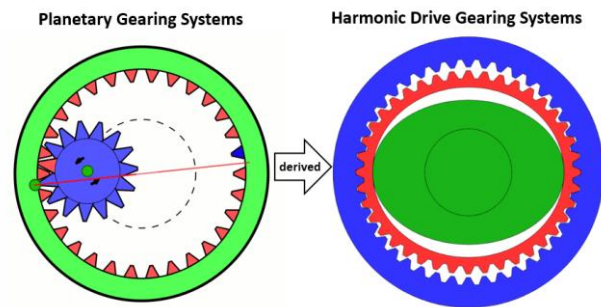

Figure 1. Drawing of a Harmonic Drive Gearing Systems

1.1 The Basic Functioning Principle

Harmonic drive gearing systems were primarily derived from special planetary gearing systems with a carrier as the main input reaction member. The output speed and torque are obtained from satellites that diverge from a central axis. The function of a harmonic drive system is, in fact, a differential gear with a front-wheel drive in which the gear is achieved by the elastic deformation of the flexible spline. A spline harmonic drive gear is typical for its unique structure called "strain cam wave generator". The principle of a function of harmonic gear systems is based on an elliptic deformation of a flexible bearing and a member of the flexible spline. Harmonic waves are generated by means of rotation on the wave generator with the flexible spline. The waves are received on the toothing of a circular spline and a flexible spline which is a movement characteristic for harmonic gear systems. Moreover, a rotation of a flexible spline is opposite to a rotation of a wave generator, see Figure 2. Thus, the input torque moment is transmitted on a wave generator and it is removed by means of holes in the back of a flexible spline.
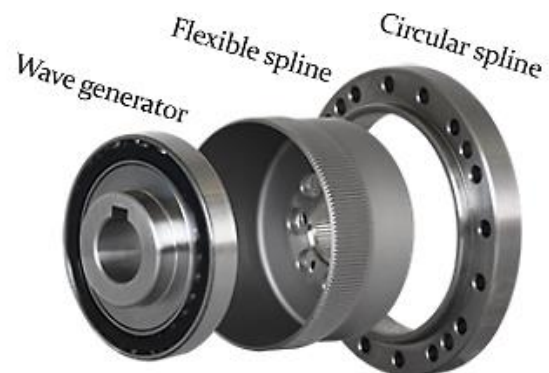

Figure 2. The Main Construction Parameters of Harmonic Gear Systems

The harmonic drive system is a toothed gear with a cam wave generator. The bottom of the flexible spline is reinforced by a flange that is used for its attachment to the output shaft of the transmission system. A circular spline with internal teeth allows harmonic drive gearing systems to be attached to a specific application of its use. The wave generator shall be mounted to an independent input shaft of a gearbox or an electric motor shaft. The structure or the connection of the wave generator of a harmonic drive gear partially eliminates inaccuracies in the bedding of the input shaft.

The structure of these unique drive gearing systems allows the transmission of enormous torque. The harmonic drive system may have really high transmission ratios due to their small size.

The gear ratio of the harmonic drive system was defined on the basis of the number of teeth of the flexible spline and a circular spline. The harmonic drive system may work as a speed reducer, a speed multiplier or a differential [Concli 2018].

1.2 The Main Characteristics of Harmonic Drive System Accuracy – Backlash and Rigidity

The principle of harmonic drive system functioning is based on the mechanism of a rotationally elliptical deformation of the flexible spline using the elliptical wave generator and a subsequent tooth generation. However, this mechanism is closely related to two sources of the non-linear behaviour of a harmonic drive system.
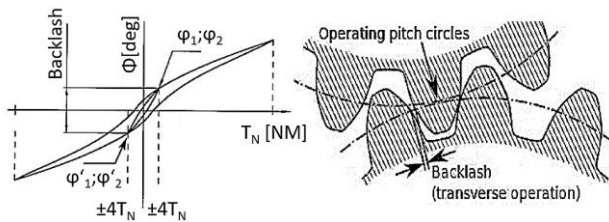
Figure 3. Backlash [Concli 2018]

The backlash is a surplus clearance between the generating teeth compared to an ideal case of generating an involute profile, where an ideal state without any clearance is considered. In practice, the backlash is achieved by an adjustment of the profile of the teeth or a modification of the axis distance between a pair of gears (Figure 3). Non-linear torsional characteristics of the flexible spline are predominantly caused by a thin-walled element construction and that is, however, necessary to allow the flexible elliptical deformation under the outer toothing.

2. THE DEVELOPMENT OF THE 3D CALCULATION MODEL

To develop a 3D calculation model, it is necessary to clearly define the area and parameters we intend to explore. This article primarily analyses the backlash of a dynamic 3D model of a harmonic gearbox. This task is rather complex, as it is a system of several contact links. Moreover, a number of those links are flexible and resilient. Even though their deformation is below the yield strength of the material, its value is in tens of millimetres. The task was divided into more parts. In the first part, we analyse the geometry of individual bodies. A rolling-element bearing is normally supplied by a producer and it is standardised, i.e., material characteristics and dimensions are known. We know the number of teeth of the bodies and a tooth meshing. Using these parameters, we can approximately define the geometry dimensions of the whole harmonic gear system.

2.1 The Description of Construction Geometry of the Bodies

Construction dimensions were acquired by a numerical calculation method. We decided to solve this issue in Matlab software. First, it was necessary to develop the geometry for a wave generator. The profile of this body is of a flattened ellipse shape. This ellipse is flattened due to a deformation of the flexible bodies of the harmonic gear system. In case of a standard elliptical wave generator geometry, a penetration would occur in the gearing, mainly during rolling. The inner diameter of the bearing was considered as a generating curve parameter; the curve was to be pressed into the bearing. In line with this parameter, the overall length of a cam generating curve was calculated (1).

$$R_{loz} = A + B \cdot \cos(2\varphi) \qquad (1)$$

$$R = A + B \cdot \cos\left\{2 \cdot \left[\frac{\pi}{2}\left(\frac{\varphi}{2\pi}\right)^n\right]\right\} \qquad (2)$$

For this calculation, it was important to find the parameters for functions A and B, so that the length of the ellipse curve was equal to the length of the bearing arc. This figure was determined numerically, according to an arc length function and an ellipse equation (2). The whole function is rather long and difficult; therefore, we will not give its details in this article. Based on this function, ellipse coordinates were determined by the equations (3) and (4):

$$X = R \cdot \cos(2\varphi) \qquad (3)$$

$$Y = R \cdot \sin(2\varphi) \qquad (4)$$

Indicated coordinates X and Y were written down to a txt. text file. This set of coordinates then generated the final elliptical shape of the wave generator, see Figure 4.
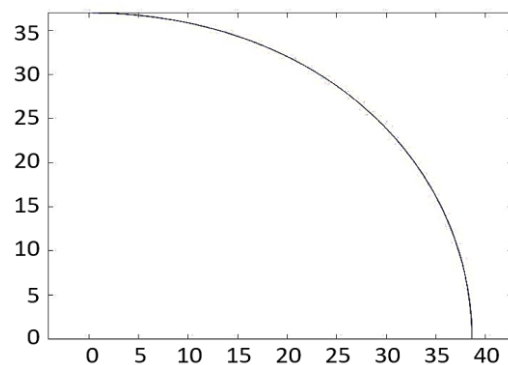
Figure 4. Ellipse Geometry Shape in MATLAB

In the following step, it was necessary to design the geometry of the flexible spline and circular spline bodies of action. We need to realise that this tooth generating does not occur on a circular trajectory as it is with conventional rigid toothed gear systems. This rolling trajectory resembles a star shape. In Matlab, a program was developed by defining classical analytical formulas to calculate the geometry of a cylindrical gearing. This program was limited by boundary conditions such as the arc length of the outer bearing ring and shifting the gear curves so that no penetration would occur in gearing. The program, just like in the case of the wave generator, wrote the coordinates of X and Y points down to a txt. text file.
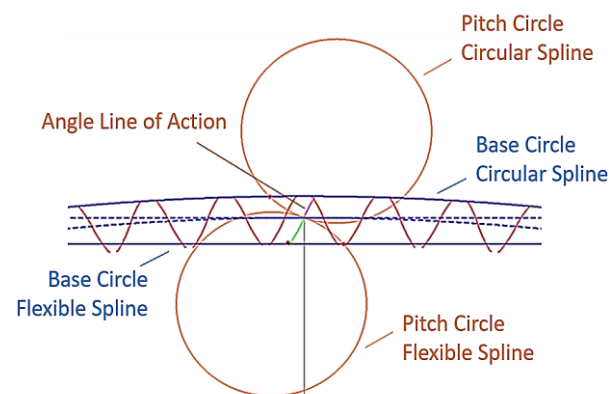
Figure 5. Gearing Geometry in MATLAB

After having loaded this file, we found the generating curves of a flexible spline gearing and a circular spline gearing. These curves were generated for 200 and 202 teeth. In Figure 5, meshing circles and gearing profiles may be seen [Daffner 2017].

2.2 The Description of the Formation of the 3D Model

In the following step, it was necessary to create a 3D-volumetric geometry of the harmonic gear system. CAD and Creo 5.0 were used to complete this task. This programme is fully parametric and geometry can be changed by an application of parameters and mathematical functions. In the first stage, text files with the geometry of wave generator bodies and both gearings were imported into Creo 5.0 sketchbook. The final 3D-volumetric shape of the harmonic gear system would be modelled from these generating curves. The bodies were reflected according to the gear system axis, see Figure 6.
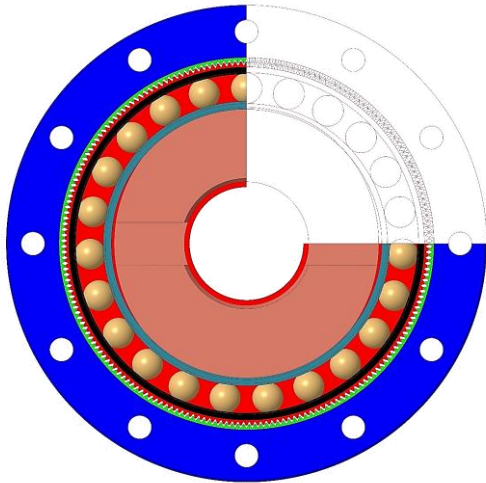
Figure 6. 3D Model of a Harmonic Gear System in CREO

In this 3D model, it is possible to parametrically change some dimensions such as a gearing module, the number of teeth of the harmonic gear system and the coefficient n for a cam (2). The parametric model is linked to parameters that were generated in Matlab. Therefore, it is possible to change parameters and regenerate the model. The regeneration will result in a new 3D shape of the whole harmonic gear system. This parametrisation is important due to a fast modification when backlash is found out and due to further research [Napau 2008].

2.3 The preparation of a 3D Model for Meshing

It is not suitable to mesh a 3D model as a whole volumetric body at once. We used Apex Grizzly by MSC for meshing. This programme enables us to use linear hexagonal elements that guarantee quality results with a relatively small number of elements.
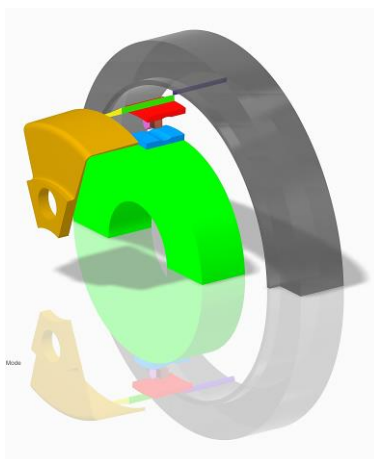

Figure 7. Preparation of a Model for Meshing in CREO

Therefore, the model had to be split into smaller symmetrical parts, see Figure 7. This model was exported to a step format.

2.4 The Meshing of the 3D Model

To achieve good results, it was needed to generate a homogeneous mesh. Therefore, the bodies were meshed individually and they were always split symmetrically according to the gear system axis. First, it was necessary to ensure an equal number of elements for both gearings. This was achieved by generating one tooth for each link and by generating a mesh of the same size and the number of elements. These teeth were then reflected around the axis for the number of 200 and 202 teeth. This reflection ensured a homogeneous mesh for all teeth, see Figure 8.
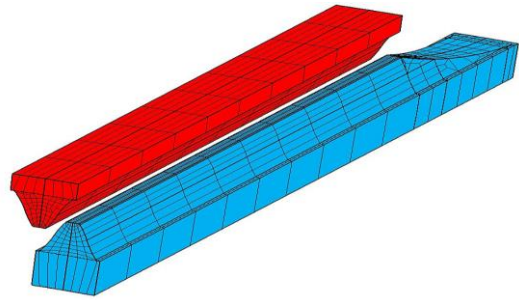

Figure 8. Detail of the Mesh for Gearing in APEX

In the following steps, the other bodies were meshed. The wave generator was split into quarter sections, whilst each one was also reflected at the axis, like the other bodies see Figure 9.
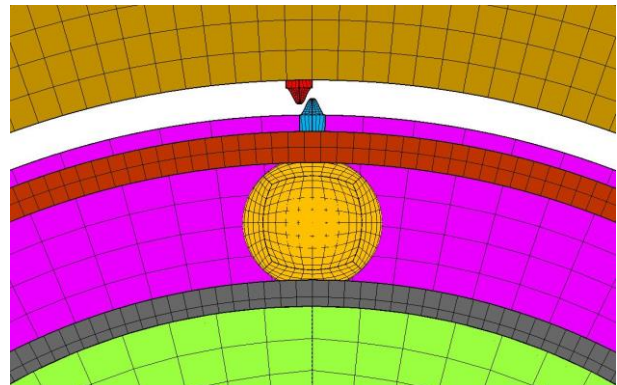

Figure 9. Detail of the 3D Mesh in Apex Grizzly in APEX

The whole harmonic gear system had 544,336 elements with a network density of 1 mm. The network density for a toothing of a circular spline was 0.5 mm. Individual links were exported one by one in bdf. format.

2.5 The Development of the Calculation Model

The simulation was completed in Marc Mentat 2018 - calculation programme. Individual bodies of the harmonic gear system were gradually imported to this programme, see Figure 10. Subsequently, these bodies were assigned material characteristics. Marc Mentat enables us to work with the analytical surface and that is nearly vital for the gearing shape and circular areas. After turning on the analytic surface function, the programme rendered the curve by nods. As a result, the programme does not consider the geometry of bodies as a sequence of nodes along the edge (step changes), which is the main issue with FEM analyses. It considers smoothed surfaces without step changes in surface shape [Belorit 2017]. The body of a flexible spline was divided into two parts as well as the body of a circular spline. The division enabled us to form

a homogenous network in the area of a toothing. Rolling elements were defined as a mutual body to simplify calculations, and each ball could roll around its axis as in a real case.
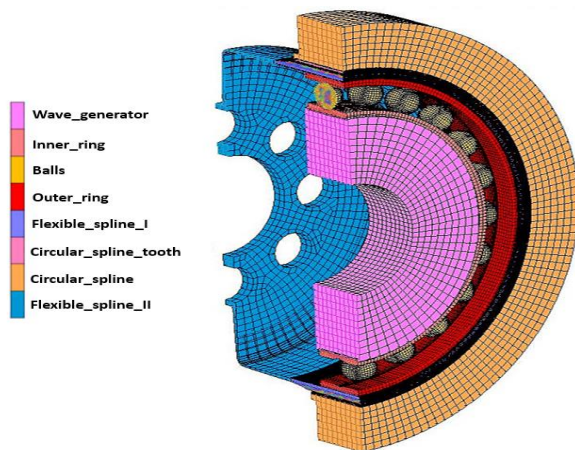

Figure 10. Defining the Contact Bodies in Marc Mentat in APEX

Marc Mentat allows an option to set the contact between the "GLUE" bodies, i.e. bodies solidly glued together, and the "TOUCHING" contact; the bodies that are in contact. It is this option of the contact of bodies which is vital for a correct assembly of a gear system. It is necessary to realise that the whole gear system was designed in an unrecreated, non-deformed shape. It is in Marc Menat that the elastic deformation of the bodies will occur, see Figure 10.
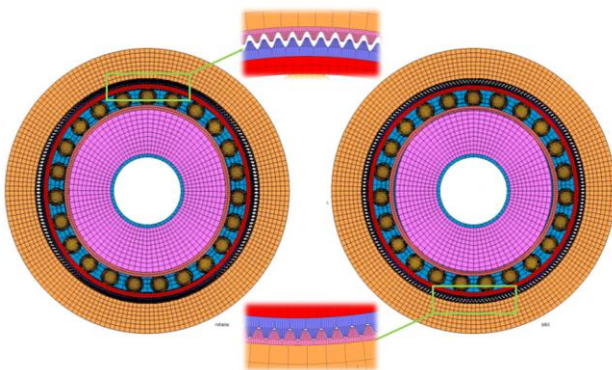

Figure 11. Deforming a Harmonic Gear System in APEX

Each contact may be set by means of contact tables, see Figure 12. The division of the contact elements and a contact type between individual elements in pressing is obvious, as seen in Figure 12 a Figure 10. Moreover, when rotated the 3D model, the contact type "TOUCHING" remained only among teeth of a circular spline and a flexible spline, the other contacts were of a type "GLUE" and, thus, elements were solidly "glued together". In these tables, it is possible to set which contact shall be turned on or off and within which step. In this way, we can assemble the whole gear system. In the first step to set it into motion and let it run for a certain time in the second step. Within this step, we can even examine the wear of the gearing and its service life; however, this article does not focus on this type of research. Within the last two steps, we let the harmonic gear system be tested against its backlash. While testing, we followed the instructions suggested by the producer of the gears. We locked the wave generator and the circular spline and we loaded the flexible spline with a torque of 100 Nm, first in the direction of rolling of the gear, then in the opposite direction. By this method, we found out the real gearing clearance and backlash [Chen 2017].

Figure 12. Defining Contact Tables in MARC

The harmonic gear system was rotated by the application of RBE2 point, which is perfectly hard and rigid point. This point was attached to the inner elements of the wave generator. Then we replaced a driving shaft. Output data was taken from RBE3 point, which is an elastic point. This point was attached to the rear part of the harmonic gear system and compensated the output shaft, see Figure 13.

A cage of a flexible bearing was substituted by a connection of individual rolling elements by means of springs of high rigidity. Thus, another flexible element that could affect the task was removed. This process proved to be as efficient as an application of a real cage (This statement is not valid in the research of a bearing, an application of a cage is efficient from the standpoint of wear of bearing rolling elements and inertia.
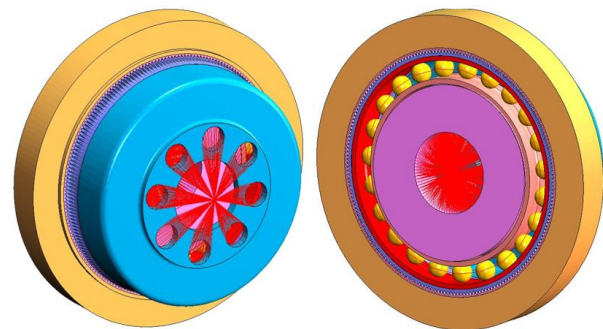

Figure 13. Attachment Boundary Conditions for Simulation in MARC

In the end, "JOB" for the whole task was created. In this "JOB", total friction, the examination of a variable, generating analytical surfaces, and switching Loadcase on and off was set. The whole task was checked within the last step, and solver was launched. Calculation time was approximately 80 hours. The harmonic gear system was pressed, rotated by 30 degrees due to a correct position of all bodies and within two steps, it was loaded with torque to examine the backlash. After the solver has successfully calculated the whole task, it was possible to proceed to the post-processing of the task.

3. DATA PROCESSING

Data processing is rather easy. A user chooses the variable they want to examine and switches on the body. In this article, we primarily focus on backlash stress and the gear system clearance with regards to the flexible spline.
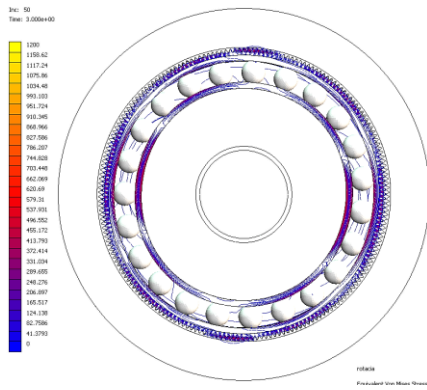
Figure 14. Data Processing in Marc Mentat Post-Processing

The first checked variable was maximum stress by Von Mises. The maximum value of this stress was 1,200 MPa in the area of rolling bodies and an outer flexible bearing ring, which is a rather real figure if we consider the Hertzian contact. It is needed to verify this stress to avoid penetration and crossing of teeth in the gear. In case of penetration of the bodies, the maximum stress would increase rapidly, reaching almost 4,000 MPa, which is not real. After the verification of the correct geometry, we analysed shifts and stress in the flexible spline. For the sake of more accurate results, this body was split into two parts - a rear part and a front part with gearing. This Splitting enabled us to mesh the rear part with a more coarse mesh and to use a finer mesh for the gearing. In the end, both parts were glued together using "GLUE" contact.
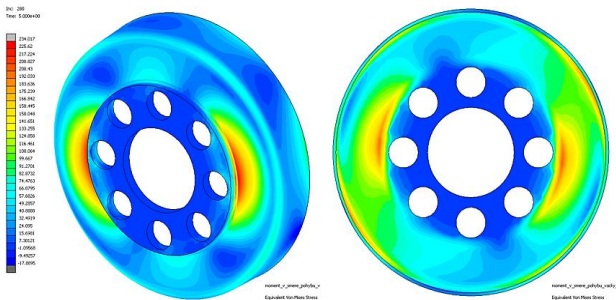

Figure 15. Stress Distribution in the Rear Part of the Flexible Spline

In Figure 15, the stress distribution is displayed when applying stress on backlash. It is the rear part of the flexible spline. Maximum stress was around 234 MPa, which is a rather good figure for this material. It means the thickness of the wall of this link was dimensioned correctly. Likewise, the front part of the flexible spline and gearing were analysed.
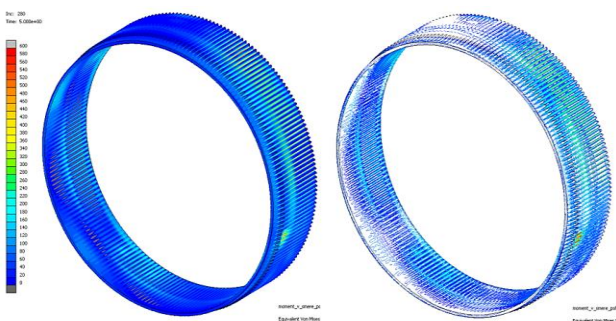

Figure 16. Stress Distribution in the Front Part of the Flexible Spline

In Figure 16, the stress distribution is displayed when applying stress on backlash regarding the front part of the flexible spline. This figure reached up to 600 MPa in contact points when rolling.

When rolling, several teeth are engaged; this caused the stress distribution among several teeth, see Figure 17 and Figure 18.
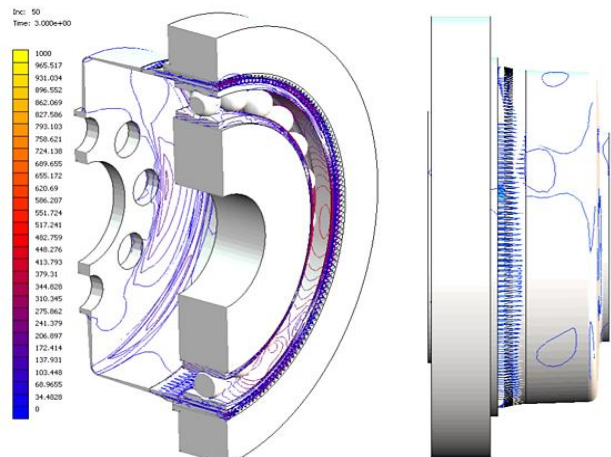

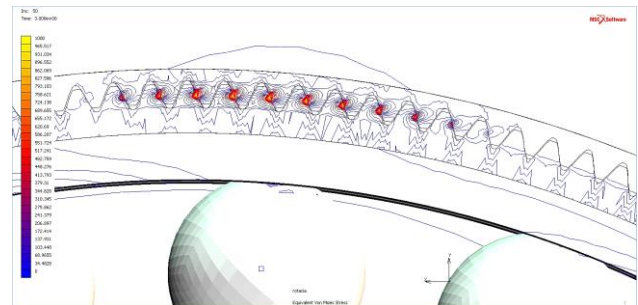**Figure 17.** Stress Distribution in the Gearing Profile

Figure 18. Detailed Stress Distribution in the Gearing

After a proper examination of the stress values regarding the flexible spline, we concluded that the task was carried out correctly. No penetration and step changes in stress were recorded. With this evaluation, we analysed the harmonic gear system backlash in the last step.

Marc Mentat enables us to display the data in a specific point, using "History plot" function. As we added RBE3 point in the task, see Figure 13, we can read some variables in this point accurately. The first analysed variable was the amount of rotation at the torque of 100 Nm. The torque approached linearly over 100 steps.
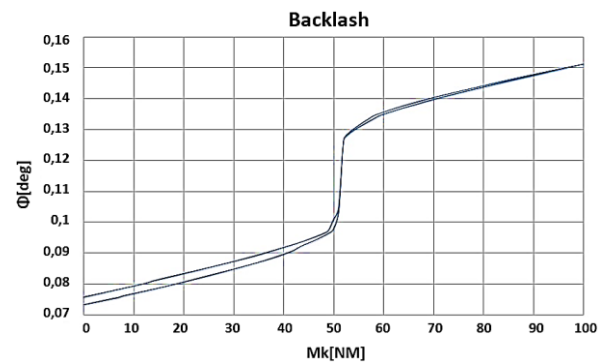

Figure 19. Evaluation of the Backlash in Excel

Figure 19 shows the course of the harmonic gear system backlash. **The amount of rotation of the flexible spline at the torque of 100 Nm and with a braked circular spline and wave generator is about 0.08°.** This rotation is due to the thin-walled structure of the flexible spline. This link is a machined forging.

The wall must be thin and elastic enough to be able to transfer elastic deformation; at the same time, it must be rigid enough to be able to transfer the torque in the gearing. Figure 20 shows the course of the reaction force that affected the RBE3 point [Zou 2013].
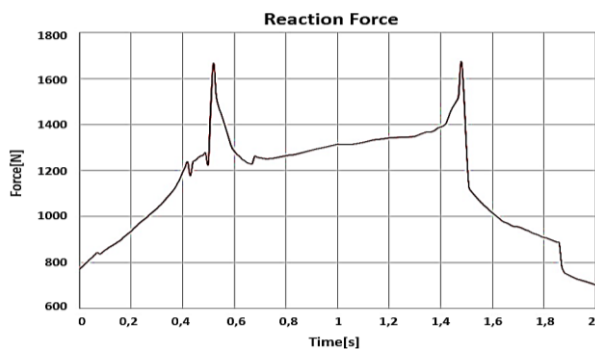

Figure 20. The Course of Reaction Force in Excel

4. CONCLUSION

In the chapters of this article, we gradually analysed the procedure of the development of the 3D calculation model of a harmonic gear system and we analysed the amount of backlash. To complete a complex task as this, several programmes have to be applied simultaneously. We recommend to start with simulations of rather simple tasks or just certain links, not the whole task at once. It is better to try out the task with a rather small model and then apply it to larger and more complicated tasks. It is necessary to emphasize that a measurement of the size of a backlash of the harmonic gear system was also performed on a real constructed system. Our 3D model is congruent with the real measurement up to 4 per cent, which confirms the accuracy and preciseness of the model. The article focuses especially on further dimensions and optimization of operational characteristics of harmonic gear systems and on a decrease of a size of the backlash. With this research, we explained the issue of harmonic gear systems and proposed the issue of their numerical solution. This model was used as a basic model for the following research, which will be focused on optimisation of design parameters for certain backlash. Our primary objective will be to optimise the parameters so that the backlash would be as small as possible. It is necessary to realise that industrial robots mostly apply these gear systems for very precise applications that require a really high rate of accuracy and rigidity. Obtained knowledge mentioned in the article is beneficial for constructions of similar parametric analytical 3D numerical models. Furthermore, a description of a process to form a 3D numerical model in the programme MSC Marc Mentant is provided. The programme is widely applied for the simulation of rolling toothed gears and it can be applied in a large area of the research of gear systems. Therefore, a description of the issue of a substitution of a cage of a rolling bearing and a definition of boundary conditions by means of points RBE2 and RBE3 will be beneficial for the design of similar models.

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