

CASE STUDY OF PRODUCTION OPTIMIZATION USING THE ENUMERATIVE METHOD

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This article examines the possibility of determining the optimal amount of inventory for an assembly line with a small number of parts. The optimization process is done using linear programming methods, specifically the enumerative method, and its results are compared by an approximation method that does not require such a number of iterative steps. The procedure of creating a purpose function defined conditions of solvability of the task and subsequently determined the total optimal size of parts stocks and also the size of stocks of individual parts before the actual start of assembly is explained. This procedure is applicable to a small number of variable parameters.

KEYWORDS

Linear programming method, enumerative method, optimal amount of inventory

1 INTRODUCTION

The primary goal of almost all manufacturing companies today is to maximize profits. There are many ways in which this goal can be achieved. Some companies prefer a form of strong marketing, others rely on product quality. By optimizing production processes, a company often finds a way to reduce certain costs or increase revenue. Optimization can be defined as a process that aims to find solutions that are "better" than the current state (already achieved or known). It is a process of improvement that may or may not result in an optimal solution. We try to travel the shortest possible way, buy the best possible product at the lowest possible price, and so on. Simply put, in our activities we try to minimize something, respectively maximize. In essence, this is about determining the extreme of one or more functions. However, optimization must respect limited resources, supplier and customer requirements, and various other limiting or limiting factors that may be caused by the market situation, human resources, production safety requirements, environmental protection, and many other factors. This is the optimization of more or less complex systems, which are affected by many more or less predictable factors. Mathematical programming methods are among the most popular and in practice the most frequently used methods for solving the optimization of complex processes.

Linear programming (LP) is a relatively small section from a wide range of different optimization methods and mathematical programming tasks. However, this is an area where many economic, technical, and mathematical problems can be solved easily and quickly. In linear programming, optimization tasks are involved when the purpose function and its boundaries are linear. Linear programming tasks are either of the maximization or minimization type, they contain m constraint conditions with n unknowns.

[Hu 2013] used the method of linear programming to predict the temperatures of some sensor nodes. The predictions were able to reduce the sampling frequency of the node and optimize the deployment of the nodes to reduce the power consumption of the sensor. [Mizuno 2016] presents a new approach to optimal energy planning using linear programming for an emergency isolated power generator installed in a large hospital. [Mahrous 2012] used the method of linear programming to minimize transport costs, in [Damian 2021] this method was used to set the optimal maintenance process to increase productivity, redistribute staff and minimize downtime, in [Milos 2019] to change to production planning in the underground mine. In order to carry out optimization studies, [Maroufmashat 2016] is considering minimizing the capital costs of hydrogen filling stations and the costs of operating and maintaining all energy nodes in the network. In addition, it examines the optimal operation of energy conversion and energy storage technologies within each node and the optimal interaction between energy nodes in the network. Production planning in an underground mine plays a key role in the mining company's business. A more efficient production optimization scheme that integrates limited optimization with decline curve analysis to predict future well production performance is addressed by [Ekkawong 2016]. The net value of the project will be reflected in the target function, which includes maximizing condensate production and minimizing wastewater production, while also respecting the daily nomination for gas production. [Olafsson 2006] used linear programming for treatment planning, testing several variants of simplex and internal point methods to solve the problem. [Sukono 2017] solves the optimization of the investment portfolio using a model of linear programming based on genetic algorithms. Portfolio risk is assumed to be measured by an absolute standard deviation and each investor has a risk tolerance for the investment portfolio. To complete the problem of optimizing the investment portfolio, the problem is arranged in a linear programming model. In addition, the determination of the optimal solution for linear programming is performed using a genetic algorithm. [Oladejo 2019] and [Veselovska 2014] also used linear programming tools to examine production costs and determine the optimal profit in bakery production. The study [Klosowski 2017] is an attempt to develop a model of a decision regulator enabling the simultaneous optimization of stocks of semi-finished products arising from cutting and punching waste. [Kira 1997] suggests that a stochastic linear programming (SLP) model be used in production planning decisions to better reflect reality and provide an excellent solution. The model remains computationally manageable despite the precise incorporation of uncertainty and the imposition of sanctions in the event of a breach of restrictions. A problem is presented that illustrates the superiority of the proposed model over those currently in use.

The study [Mahmoudi 2019] is an attempt to propose a new method of solving problems using grey linear programming. The proposed methods of grey linear programming suffer from many disadvantages, such as the weakness of the solution of linear programming with grey numbers in constraints, inappropriate results with a lower limit if it is greater than the upper limit, solutions outside the feasible areas, etc. To overcome these disadvantages, the current study proposes a new method of linear programming with grey parameters. Later, its solutions are compared with other alternative methods. Comparative analyses of the methods revealed the superiority of the proposed method compared to other

methods in terms of the quality of the optimal solution and computational steps. Unlike other existing methods, which usually consist of several stages, the proposed method has only three phases to solve grey linear programming problems and is therefore easier to work with. Nonlinear programming and optimization methods [Mostafa 2016] provide a promising method for pore network modelling when computed tomography imaging may not be readily available. The authors in [Kulka 2017] used heuristic optimization to solve the transport network in a large city. [Izaz 2011] used sensitivity analyses for linear programming techniques to maximize the profit generated from ICI production models in Pakistan. In [Fabianova 2017] is use of software applications and computer simulation for more effective quality management. Simulation tools offer incorporating the variability of more variables in experiments and evaluating their common impact on the final output. [Onofrejova 2021] follows the idea of continuous digitization and transfer to digital factory. Use a reliable miniaturization technology to monitor the working environment to ensure appropriate working conditions.

Optimization methods are especially suitable for solving classical linear programming problems, which are given by a purpose function and constraint conditions. In this way, it is also possible to solve some integer programming tasks, which are specified as linear problems. However, the values of the variables in such tasks must satisfy the strict integrality condition. Nevertheless, there are also tasks for which no solution can be found using linear programming. In this case, it is appropriate to choose another optimization method that will be able to solve the problem more broadly.

Integer linear programming differs from general linear programming only in that all or only some variables can only acquire integer values [18]. Integer programming tasks are usually more difficult to solve than tasks without the condition of integrity. If the optimal solution is not an integer, there are two options:

1. Round the non-integer solution to the nearest lower integer values and correct the value of the purpose function;
2. Apply exact methods of integer programming and determine the optimal solution from the set of basic admissible solutions in the set of integers.

The first option is the simplest, but it does not always lead to a correct result. We can use it when the fractional (non-integer) part of the solution is small (statistically insignificant) compared to its integer part. Depending on the nature, the methods for integer solution of LP problems can be divided into:

1. Empirical methods - the most commonly used method is integer rounding or so-called enumerative method (brute force method).
2. Exact methods - most often based on the principle of determining the so-called cutting surfaces or combinatorial processes, e.g. Branch and Bound method.
3. Special methods - for solving problems with a specific structure or some heuristic algorithms, which for otherwise exactly unsolvable problems can determine at least a solution close to the optimum sought.

The enumerative method (brute force method) consists in laboriously testing all the possibilities of "integerization" of a non-integer solution. It is suitable if the set of admissible solutions to the problem is limited, i. the whole is contained in a certain finite (multidimensional) space. It is then sufficient to find an interval for each variable x_i , where for all admissible

solutions the variable x_i certainly lies in this interval. Subsequently, all points of the block with integer coordinates are successively tested, the validity of the constraint conditions is verified for each of them, and if the point thus defined is an admissible solution, the value of the objective function is calculated and compared with the best solution found so far. The big disadvantage of the method is its computational complexity.

We will show the procedure of the solution using the enumerative method on the example of an assembly line on which four parts will be produced. We will try to explain the process of creating a purpose function, define the conditions of solvability of the task and determine the optimal size of stocks of individual parts as well as the size of stocks of individual parts before the actual start of assembly.

2 MATHEMATICAL MODEL CREATION

A total of m types of parts is produced on the line. The productivity of the line, which we measure in the number of pieces of the i -th part ($i = 1, 2, \dots, m$), is p_i pieces / day. Changing the line settings when switching from the i -th part to the j -th part and vice versa takes s_{ij} days (hours). The average consumption of parts of the i -th type is r_i pieces / day. Even consumption is assumed. Our requirement is that a production plan be drawn up to ensure a minimum consumption of individual parts and a minimum average stock value. The average inventory will be determined from the average inventory of individual parts according to the formula (1):

$$z = \sum_{i=1}^m c_i \cdot z_i \tag{1}$$

- z_i - average stock of the i -th part in pieces,
- c_i - conversion factor for parts of the i -th type,
- z - average total stocks.

The conversion factors c_i can be chosen so that the value of z represents the volume of committed funds or storage space requirements.

A necessary and sufficient condition for the solvability of a given task can be written in the form of a capacity condition (2):

$$\sum_{i=1}^m \frac{r_i}{p_i} < 1 \tag{2}$$

The entered optimization criterion will lead to the fact that all the remaining time after ensuring the average daily consumption of all types of parts will be used to rebuild the line. Assume that a planning cycle of T days (e.g. 1 year) is selected. During this cycle, each type of part must be put into production at least once. If we choose the batch Q_i and the number of batches entered during one planning period of the cycle x_i ($i = 1, 2, \dots, m$) for entering parts of the i -th type, this choice must satisfy the relation (3):

$$T \cdot r_i = Q_i \cdot x_i, \quad (i=1,2,\dots, m) \tag{3}$$

From there, after editing, we get (4):

$$Q_i = \frac{T \cdot r_i}{x_i} \quad (i=1,2,\dots, m). \quad (4)$$

In the considered case, it is possible to achieve the smallest amount of average stocks of individual types of parts when they are regularly entered into production. This corresponds to (5):

$$z_i = Q_i/2, \quad (i=1,2,\dots, m). \quad (5)$$

After substitution (4) into equation (5) we receive (6):

$$z_i = \frac{T \cdot r_i}{2 \cdot x_i} \quad (i=1,2,\dots, m). \quad (6)$$

The average amount of stocks in total is then given by:

$$z = \frac{T}{2} \cdot \sum_{i=1}^m \frac{c_i \cdot r_i}{x_i} \quad (7)$$

When regularly entering three types of parts into production (marked e.g. A, B, C), it is possible to arrange them in a total of four ways:

A, B, C,	(A, B, C)	A, ...
A, B, A, C,	(A, B, A, C)	A, ...
B, A, B, C,	(B, A, B, C)	B, ...
C, A, C, B,	(C, A, C, B)	C, ...

These sequences are, as can be seen, periodic. In the first group A, B, C is repeated, in the second group A, B, A, C, in the third group B, A, B, C and in the fourth group C, A, C, B.

For the first sequence, it is then:

$$x_1 = 1, \quad x_2 = 1, \quad x_3 = 1$$

for the second sequence is:

$$x_1 = 2, \quad x_2 = 1, \quad x_3 = 1$$

for the third sequence is:

$$x_1 = 1, \quad x_2 = 2, \quad x_3 = 1$$

for the fourth sequence is:

$$x_1 = 1, \quad x_2 = 2, \quad x_3 = 2.$$

If we generalize the procedure used for three types of parts, we get to the following rule for creating groups:

- we will select n types of parts from all m types of parts in any way and, regardless of the order of them, we will compile one subgroup;
- a similar subgroup is then assembled from the remaining (m-n) parts;
- we then compile one group from both subgroups by inserting the whole first subgroup before each element of the second subgroup;
- the number n can be selected in the interval <0, m-2>;
- the total number of all possible alternations is given by (8):

$$\sum_{n=0}^{m-2} \binom{m}{n} \quad (8)$$

Number combinations $\binom{m}{n}$ to determine the total number of all possible ways of alternating batches of parts can be taken from the well-known Pascal's triangle (Fig.1):

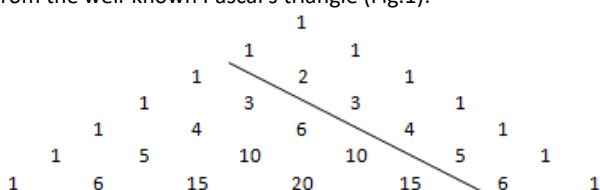


Figure 1. Pascal's triangle

For example, for three product types, the total number of rotation modes is 1 + 3 = 4, for four product types 1 + 4 + 6 = 11 rotation modes.

The following x_i values correspond to the above rule for creating recurring groups:

$$x_i = \begin{cases} m-n & \text{for first subgroup} \\ 1 & \text{for second subgroup} \end{cases} \quad (9)$$

It will be appreciated that the choice of the length of the planning cycle T and the choice of the method of alternating the batches of parts will be appropriate to make independently of each other. This is, after all, also apparent from the shape of the criterion function (7).

Suppose we have already chosen some method of alternating batches of parts and, by calculating the appropriate values, they have determined the corresponding time for the adjustment of line S during the planning cycle T.

The optimal length of the planning cycle T can be determined from the capacity condition (10):

$$\frac{S}{T} + \sum_{i=1}^m \frac{r_i}{p_i} = 1 \quad (10)$$

From here we get:

$$T = \frac{S}{1 - \sum_{i=1}^m \frac{r_i}{p_i}} \quad (11)$$

This corresponds to the value of the criteria function:

$$z = \frac{S/2}{1 - \sum_{i=1}^m \frac{r_i}{p_i}} \cdot \sum_{i=1}^m \frac{c_i \cdot r_i}{x_i} \quad (12)$$

Since the value of S and the values of x_i ($i = 1, 2, \dots, m$) depend only on the chosen method of alternation, we can recommend enumerative as the most suitable method for minimizing the value of z. The use of this method consists in finding and evaluating all possible variants of rotation. Obviously, the iterative method is only suitable for a small number of different types of parts. If we are satisfied with the approximate solution, then we can proceed as follows:

- we classify all types of parts in descending order (from the largest to the smallest) according to the value of the product $c_i \cdot r_i$;
- classified in this way, we divide them into two subgroups so that the first subgroup will be the first type of parts;
- we will evaluate the corresponding variant by alternating batches of parts;
- we will extend the first subgroup with another type of parts in order, etc.;
- the process ends as soon as only one type of parts remains in the second subgroup.

According to the above approximation procedure, it will be necessary to evaluate only m-1 variants of alternating batches of parts. For example, for 4 types of parts, using the enumerative method, we would have to evaluate a total of 11 variants, while with the approximation method, only 3 variants.

3 EXAMPLE

The line produces 4 types of parts:
part A in the number $p_1 = 150$ pcs / day

part B in the number $p_2 = 150$ pcs / day
 part C in the number $p_3 = 132$ pcs / day
 part D in the number $p_4 = 130$ pcs / day.
 The times required to change the line are as follows: $s_{12} = s_{13} = s_{14} = s_{23} = s_{24} = 4$ days, $s_{34} = 2$ days.

The average daily consumption of parts for assembly is:

$$r_1 = 26 \text{ pcs / day}$$

$$r_2 = 8 \text{ pcs / day}$$

$$r_3 = 10 \text{ pcs / day}$$

$$r_4 = 34 \text{ pcs / day.}$$

We will calculate the average amount of stocks in the number of pieces. We therefore elect

$$c_1 = c_2 = c_3 = c_4 = 1$$

Before starting the calculation, we still need to check whether the task is consistent:

$$\sum_{i=1}^4 \frac{r_i}{p_i} = \frac{26}{150} + \frac{8}{150} + \frac{10}{132} + \frac{34}{130} = 0,564 < 1$$

... condition satisfies.

$$1 - \sum_{i=1}^4 \frac{r_i}{p_i} = 0,436$$

..... part of the capacity that can be used to rebuild the line.

When solving the task of optimizing the average amount of stocks using the enumerative method, we must find and evaluate a total of 11 variants of alternating batches of parts on the line:

1. $x_1 = 1, \quad x_2 = 1, \quad x_3 = 1, \quad x_4 = 1,$
 $s = 14$ days, $z = 1252$ pieces, $T = 32$ days
 The order must be followed during the rotation: 1, 2, 3, 4, (1, 2, 3, 4), 1, ...
2. $x_1 = 3, \quad x_2 = x_3 = x_4 = 1,$
 $s = 24$ days, $z = 1670$ pieces, $T = 55$ days
 The order must be followed during the rotation: 1, 2, 1, 3, 1, 4, (1, 2, 1, 3, 1, 4), 1, ...
3. $x_1 = 1, \quad x_2 = 3, \quad x_3 = x_4 = 1,$
 $s = 24$ days, $z = 2000$ pieces, $T = 55$ days
 The order must be followed during the rotation: 2, 1, 2, 3, 2, 4, (2, 1, 2, 3, 2, 4), 2, ...
4. $x_1 = x_2 = 1, \quad x_3 = 3, \quad x_4 = 1,$
 $s = 20$ days, $z = 1636$ pieces, $T = 46$ days
 The order must be followed during the rotation: 3, 1, 3, 2, 3, 4, (3, 1, 3, 2, 3, 4), 3, ...
5. $x_1 = x_2 = x_3 = 1, \quad x_4 = 3,$
 $s = 20$ days, $z = 1269$ pieces, $T = 46$ days
 The order must be followed during the rotation: 4, 1, 4, 2, 4, 3, (4, 1, 4, 2, 4, 3), 4, ...
6. $x_1 = x_2 = 2, \quad x_3 = x_4 = 1,$
 $s = 24$ days, $z = 1679$ pieces, $T = 55$ days
 The order must be followed during the rotation: 1, 2, 3, 1, 2, 4, (1, 2, 3, 1, 2, 4), 1, ...

7. $x_1 = 2, \quad x_2 = 1, \quad x_3 = 2, \quad x_4 = 1,$
 $s = 22$ days, $z = 1741$ pieces, $T = 50$ days
 The order must be followed during the rotation: 1, 3, 2, 1, 3, 4, (1, 3, 2, 1, 3, 4), 1, ...
8. $x_1 = 2, \quad x_2 = 1, \quad x_3 = 1, \quad x_4 = 2,$
 $s = 22$ days, $z = 1211$ pieces, $T = 50$ days
 The order must be followed during the rotation: 1, 4, 2, 1, 4, 3, (1, 4, 2, 1, 4, 3), 1, ...
9. $x_1 = 1, \quad x_2 = x_3 = 2, \quad x_4 = 1,$
 $s = 22$ days, $z = 1741$ pieces, $T = 50$ days
 The order must be followed during the rotation: 2, 3, 1, 2, 3, 4, (2, 3, 1, 2, 3, 4), 2, ...
10. $x_1 = 1, \quad x_2 = 2, \quad x_3 = 1, \quad x_4 = 2,$
 $s = 22$ days, $z = 1438$ pieces, $T = 50$ days
 The order must be followed during the rotation: 2, 4, 1, 2, 4, 3, (2, 4, 1, 2, 4, 3), 2, ...
11. $x_1 = x_2 = 1, \quad x_3 = x_4 = 2,$
 $s = 20$ days, $z = 1284$ pieces, $T = 46$ days
 The order must be followed during the rotation: 3, 4, 1, 3, 4, 2, (3, 4, 1, 3, 4, 2), 3, ...

The order given in the individual rotation variants is binding for the rotation of whole groups. Within these groups, only the order of batches of parts no. 3 and no. 4 (other time needed to change the line).

The best solution gives the eighth variant, which is characterized by data: $s = 22$ days, $z = 1211$ pieces, $T = 50$ days. The following batch sizes of the individual types of parts correspond to this variant:

$$Q_1 = T \cdot \frac{r_1}{x_1} = 50 \cdot \frac{26}{2} = 650 \text{ pcs}$$

$$Q_2 = T \cdot \frac{r_2}{x_2} = 50 \cdot \frac{8}{1} = 400 \text{ pcs}$$

$$Q_3 = T \cdot \frac{r_3}{x_3} = 50 \cdot \frac{10}{1} = 500 \text{ pcs}$$

$$Q_4 = T \cdot \frac{r_4}{x_4} = 50 \cdot \frac{34}{2} = 850 \text{ pcs}$$

We remind you again that the order of these doses must be in order 1, 4, 2, 1, 4, 3, 1, 4, 2, 1, 4, 3, 1, ...

Using the approximation method, out of the 11 alternating variants, it would be necessary to search for and evaluate only three variants - the fifth, the eighth and the first, in that order. The optimal solution would then be identical to the optimal solution found by the enumerative method.

The proposed mathematical model solves the problem of steady state of the production process. For the application of the found solution, it is necessary to determine the initial state of stocks of individual types of parts v_i . The values of v_i ($i = 1, 2, \dots, m$) can be determined from the condition of ensuring the consumption of parts of the i -th type until the production of their first batch is started.

$$v_i = r_i \cdot \left(\sum_{0 < k_j < k_i} \frac{Q_j}{p_j} + \sum_{0 < k_j < k_i} s_{(k_j-1)^{-1}k_j^{-1}} \right) \quad i = 1, 2, \dots, m \quad (13)$$

k_i - the order of the batch of parts of the i -th species in the recurring group of batches of parts
of all kinds,

$(k_i)^{-1}$ = i - designation of the type of part whose batch is entered last in the recurring group of batches
of parts,

$(0)^{-1}$ - designation of the type of part whose batch is entered last in the recurring group of batches
of parts.

As an example, we will present the calculation of initial stock levels to implement the optimal solution designed for 8 variants of rotation 1, 4, 2, 1, 4, 3, 1, 4, 2, 1, 4, 3, 1, ...

$$v_1 = 26.4 = 104 \text{ pieces}$$

$$v_2 = 8 \cdot \left(\frac{650}{150} + \frac{850}{130} + 4 + 4 + 4 \right) = 183 \text{ pieces}$$

$$v_3 = 10 \cdot \left(\frac{598}{150} + \frac{782}{130} + \frac{368}{150} + \frac{598}{150} + \frac{782}{130} + 4 + 4 + 4 + 4 + 4 + 2 \right) = 464 \text{ pieces}$$

$$v_4 = 34 \cdot \left(\frac{650}{150} + 4 + 4 \right) = 419 \text{ pieces}$$

The total initial stock is then 1170 pieces.

4 CONCLUSIONS

The procedure of optimizing the stock size for the assembly line by the method of linear programming, specifically by the enumerative method, is explained. The procedure of creating a criterion function is explained, as well as the limiting conditions and the control of the solvability of the task. The proposed procedure is verified on the example of an assembly line on which four different parts are manufactured and the line needs to be rebuilt before assembly. The optimal stock size of the individual parts as a whole and also the stock size of the individual parts before assembly itself are determined. An enumerative method (brute force method) consists in laborious testing of all possible solutions. It is suitable if the set of admissible solutions of the problem is limited, i. j. the whole is contained in a certain finite (multidimensional) space. The big disadvantage of the method is its computational complexity, which increases with the number of variables. For three variables there are 4 combinations, for 4 variables it is 11 combinations (our case), for 5 variables 26 combinations, for 6 variables it is a total of 57 possible combinations (according to Pascal's triangle). For a larger number of variables, it is then more appropriate to use the approximation method. Modern trends and powerful computers currently offer simulation procedures for solving such tasks, which can operatively respond to changes in individual parameters in the created simulation model.

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