

# DETERMINATION OF STIFFNESS AND DAMPING PARAMETERS FROM A VEHICLE MODEL IN A FRONTAL IMPACT INTO THE RIGID BARRIER USING THE MATLAB

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The main goal of this paper is to create a computer crash model (up to the level of the double mass-spring-damper model, which is described by a system of two differential equations and where the stiffness and damping coefficients - physical parameters are calculated using the Matlab function  $Y = \text{solve}(\text{eqns})$  - Equations and systems solver) using measured real data obtained from a frontal impact of a vehicle into a rigid barrier. The vehicle crash modelling process can be performed in two ways. One of them, which is also applied in this article, is based on the System Identification Toolbox, which contains MATLAB® functions, Simulink® blocks and a special application for compiling models of dynamic systems from measured I / O data.

## KEYWORDS

Matlab, System identification toolbox, State-space model, Transfer function model, A Two Mass-Spring-Damper (2-MSD) model .

## 1 INTRODUCTION

Vehicle crash test is usually done in order to ensure safe design standards of a car software. Simulated crash tests can be performed and evaluated to the full extent of a real crash tests. This means that due to simulated crash tests it is possible to significantly reduce the costs needed for performing real crash tests with real cars. Identification system concerns construction and validation of mathematical models from dynamic input-output data [Mascenik 2014, Saga 2020]. In experiments the system reveals information about itself in terms of input and output measurements [Munyazikwiye 2013, Bozek 2021]. Several solutions based on experimental test procedures are available for identification of mathematical models [Hlavac 2018]. One of the most convenient and accessible solution is to use the System identification toolbox [Mathworks R2014a]. This toolbox is largely based on the works of Ljung [Ljung 1994] and it implements several common techniques used in the process of system identification. System identification toolbox enables us to create and use models of dynamic systems. We can use input-output data of the time and frequency domain to identify continuous-time and discrete-time transfer functions, process models, and state-space models. In a dynamic system,

the values of the output signals depend on both, the instantaneous values of its input signals and also on the former behaviour of the system. Models of dynamic systems are typically described by differential equations, transfer functions, state-space equations, and pole-zero-gain models. System Identification requires a model structure [Marcinkova 2013, Trojanova 2021].

## 2 A TWO MASS-SPRING-DAMPER (2-MSD) MODEL

The 2-MSD model (shown in Fig. 1 and used in this article) simulates a frontal impact of the vehicle into a rigid barrier. Parameters of  $m_1$  and  $m_2$  represent the frame rail- (chassis) and passenger compartment- masses, respectively. In other cases,  $m_1$  may represent the vehicle structure with energy absorbers (spring and damper), and  $m_2$ , could also be the torso with a restraint system of spring ( $k_2$ ) and damper ( $c_2$ ).

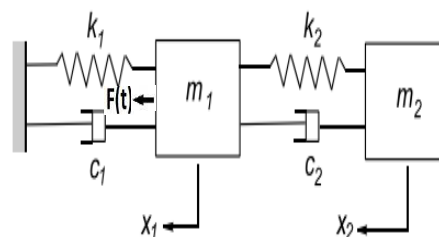


Figure 1. A Two Mass-Spring-Damper model [Huang 2002]

$k_{1,2}$  - springs stiffness coefficients

$c_{1,2}$  - damping coefficients

$x_{1,2}$  - positions of masses  $m_{1,2}$

The method for solving the impact responses of the two masses is adapted from the method used in the free vibration analysis of a two-degree of freedom damped system [Huang 2002]. The equations of motion (EOM) of the 2-MSD model are shown in Eq. (3).

## 3 GETTING DATA FROM THE IMPACT TEST

Data for the System identification Toolbox were obtained from a frontal impact test into the rigid barrier with full coverage at the speed of 56.17 kph (15.6 mps) according to NCAP (New Car Assessment Program) [NCAP 2017]. The rigid barrier also contains force sensors. Honda Civic XL 2-door Coupe (Fig. 2) was chosen for providing the impact test. A measurement recording has been processed from an accelerometer that was firmly connected to the vehicle floor at the rear of the bodywork [Vlk 2003]. Each impact test recording coming from accelerometer must be filtered by the CFC 60 filter (Channel Frequency Class) due to significant signal oscillations [Cichos 2006]. This type of filter is characterized by the following filter parameters (Tab. 1): 3-dB limit frequency, 100 Hz. Stop damping, -30 dB. Sampling frequency, at least 600 Hz. It was designed primarily for measuring deceleration on car bodies. In order to perform the impact tests correctly, signal processing must be done under specific predetermined conditions. These standard conditions are described in SAE J211-1: Instrumentation for Impact Test, Part 1, Electronic Instrumentation [Coufal 2012, Evin 2016].



Figure 2. Pre-Test Left View of Test Vehicle Honda Civic XL 2 door Coupe by Ncap [Ncap 2017]

Filter type	Filter parameters		Usage of the filter
CFC 60	3 dB limit frequency	100 Hz	Structure deceleration
	Stop damping	-30 dB	
	Sampling frequency	At least 600 Hz	

Table 1. Type of filter

#### 4 ACHIEVED RESULTS AND THEIR DISCUSSION

In Simulink, which is a graphical programming environment for modelling, simulation and analysis of dynamic systems, a custom model was created. Its main goal is to process the measured data (unfiltered deceleration signal) so that the output (in this model it is a block Scope) monitored the deformation of the vehicle – displacement signal (Honda Civic XL). In the already mentioned tailor-made model, there is a need to filter out the noisy signal that we get from the accelerometer firmly connected to the floor of the vehicle. This suitable filter is a low pass filter CFC 60 (in the model it is a Lowpass filter block), where the parameters are set as shown in the Fig. 3.

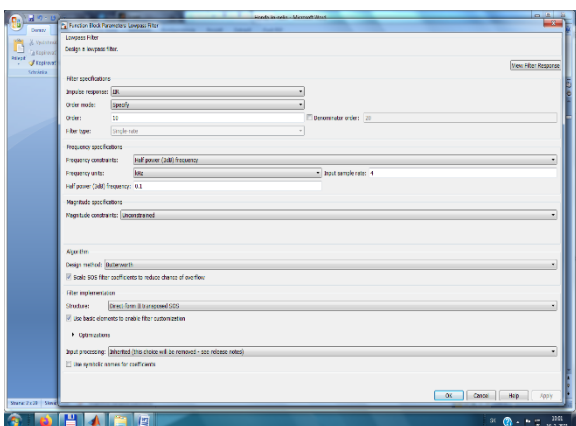


Figure 3. Parameters of lowpass filter CFC 60

The measured deceleration thus filtered must be integrated twice more in order to obtain the measured displacement

signal. The processed data from Simulink's own model, were further imported into the system identification toolbox (input-deceleration output-deformation). And from there we can obtain a transfer function of the 4th order, the shape of which is as follows:

$$tf2 = \frac{-76.15s+466.2}{s^4+35.71s^3+6024s^2+1.85e05s+4.146e05} \quad (1)$$

Parameterization of the transfer function tf2: Number of poles: 4, Number of zeros: 1 (Fig. 4), Number of free coefficients: 6 .

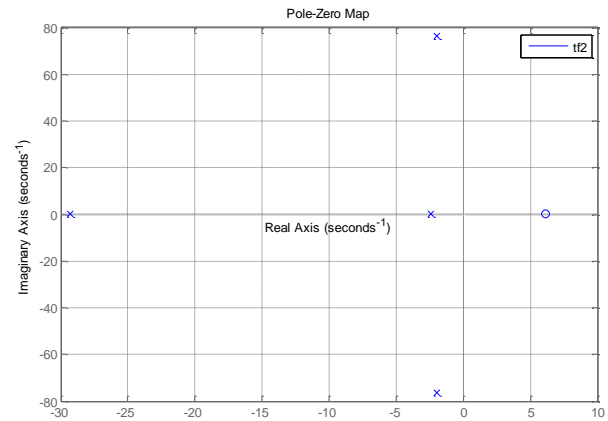


Figure 4. Pole-Zero map

Termination condition: Maximum number of iterations reached.

Number of iterations: 20

Number of function evaluations: 117

Estimated using TFEST on time domain data "Honda civic XL".

% Transfer function estimation in Matlab

Options = tfestOptions;

Options.Display = 'on';

tf2 = tfest(honda, 4, 1, Options)

Fig. 5 shows measured (red line) and estimated, simulated (blue line) outputs.

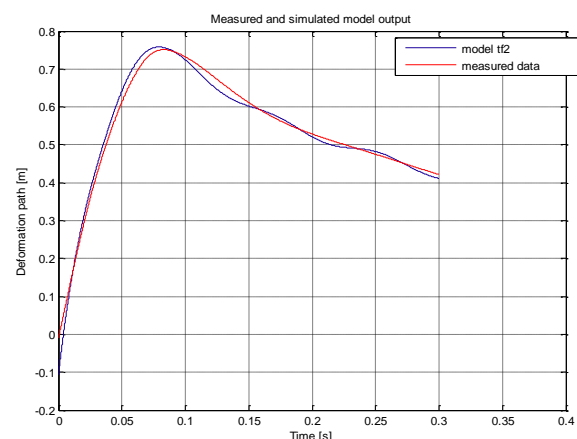


Figure 5. Measured and simulated model outputs

The transfer function tf2 corresponds to the estimated state-space model of order 4, which through the following instructions in Matlab:

num = [- 76.15 466.2];

den = [1 35.7 6024 1.85e05 4.146e05];

[A, B, C, D] = tf2ss (num, den)

takes the form:

$$\mathbf{A} = \begin{bmatrix} 1.0e+05 & & & \\ -0.000357 & -0.06024 & -1.8500 & -4.1460 \\ 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}, \quad (2)$$

$$\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & -76.1500 & 466.2000 \end{bmatrix}, \quad \mathbf{D} = 0.$$

If the frontal impact of a vehicle (Honda) into a rigid barrier is represented by the 2-MSD model (Fig. 1), then we can describe it by a system of differential equations in the form:

$$\begin{aligned} m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + (k_1 + k_2)x_1 - c_2 \dot{x}_2 - k_2 x_2 &= F(t) \\ m_2 \ddot{x}_2 - c_2 \dot{x}_1 + c_2 \dot{x}_2 + k_2 x_2 - k_2 x_1 &= 0 \end{aligned} \quad (3)$$

If we convert the system of differential equations (3) into the Laplace transform, we can derive the transfer function between  $F(t)$  - the acting force on  $m_1$  (or deceleration), and the position of the cabin  $x_2(t)$ , which we write in the form:

$$\begin{aligned} 2\_MSD(s) &= Num(s)/Denum(s) \quad (4) \\ \text{where} \\ Num(s) &= c_2 s + k_2 \\ Denum(s) &= m_1 m_2 s^4 - (m_1 c_2 + m_2 (c_1 + c_2)) s^3 \\ &\quad + (m_1 k_2 + m_2 (k_1 + k_2) + c_1 c_2) s^2 \\ &\quad + (c_1 k_2 + c_2 k_1) s + k_1 k_2 \end{aligned}$$

By comparing the coefficients of the transfer function (see Equation 1), which are obtained from experimental data using the Identification System, and the coefficients of the transfer function  $2\_MSD(s)$  (see Equation 4) at the respective power places of the numerator  $Num(s)$  or denominator  $Denum(s)$ , we can get a set of equations (5), which is used to determine the parameters  $c_1, c_2, k_1, k_2$ . (Or if we transform the transfer function model into a state-space model of order 4 - the obtained state matrix  $\mathbf{A}$  (Equation 2) and compare it with the matrix  $\mathbf{Ac}$  in the so-called canonical state-space form, the comparison of the corresponding positions will also lead to a system of equations (5)) [Munyahikwiye 2014]:

$$\begin{aligned} m_1 c_2 + m_2 c_1 + m_2 c_2 &= m_1 m_2 * 35.7 \\ m_1 k_2 + m_2 k_1 + m_2 k_2 + c_1 c_2 &= m_1 m_2 * 6024 \\ c_1 k_2 + c_2 k_1 &= m_1 m_2 * 185000 \\ k_1 k_2 &= m_1 m_2 * 414600 \end{aligned} \quad (5)$$

We can solve this system of equations in Matlab.

**General solution:**

$Y = \text{solve}(\text{eqns}, \text{vars}, \text{Name}, \text{Value})$

$[y1, \dots, yN] = \text{solve}(\text{eqns}, \text{vars}, \text{Name}, \text{Value})$

This MATLAB function solves the equations  $eqns$  for the default variables determined by  $\text{symvar}$ .

For each parameter there are 4 solutions, two real and two complex. For the physics system, only real values make sense, we do not take complex into account.

**Specific solution for the case:**

Total vehicle weight  $m_t = 1422$  kg, chassis weight  $m_1 = 1/5 * m_t = 284.4$  kg and cab weight  $m_2 = 4/5 * m_t = 1137.6$  kg

Clear

$\text{syms } c1 \ c2 \ k1 \ k2$

```
[c1,k1,c2,k2] = solve(284.4*c2 + 1137.6*c1 + 1137.6*c2 ==
284.6*1137.6*35.7, 284.4*k2 + 1137.6*k1 + 1137.6*k2 +
c1*c2 == 284.4*1137.6*6024, c1*k2 + c2*k1 ==
284.4*1137.6*185000, k1*k2 == 284.4*1137.6*414600)
```

The solution found for this case: There are two real roots for each parameter and two complex ones are neglected

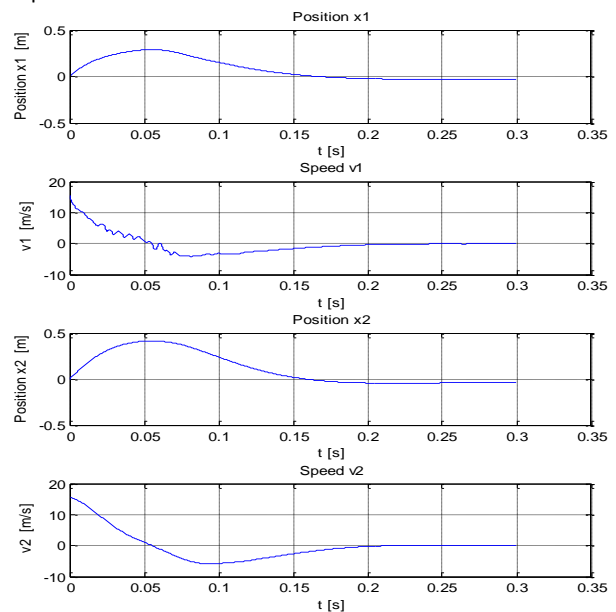
$$c_1 = 34435.783 \quad k_1 = 19420.450$$

$$-24275.563 \quad 27548.626$$

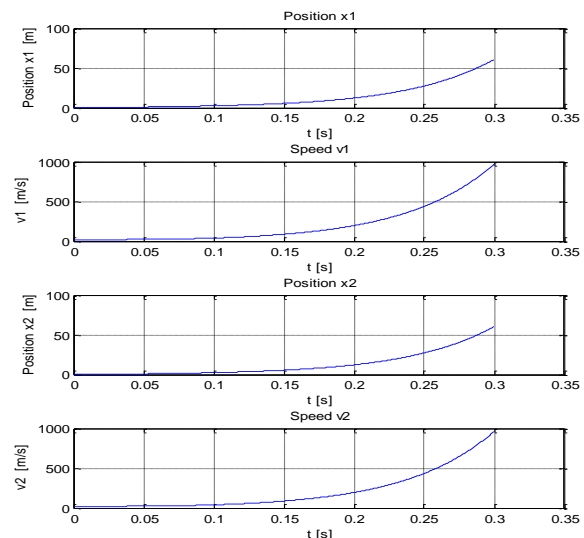
$$c_2 = 75332.067 \quad k_2 = 1780609.088$$

$$2225761.360 \quad 60265.654$$

For the case  $c_1 = 34435.783$  Ns / m,  $k_1 = -19420.45$  N / m,  $c_2 = 75332.06$  Ns / m,  $k_2 = 1780609.08$  we get the simulation result (see Fig. 6). From this figure, we can see that the dynamic crush (understand maximum deformation) of the chassis  $m_1$  is 0.3 m and is achieved in 0.05 s. For passenger compartment  $m_2$ , the dynamic crush is 0.4 m and is also achieved in 0.05 s (measured data show that the dynamic crush is 0.75 m and is achieved in 0.08 s (see Fig. 5)). This model does not fully represent the crash scenario.



**Figure 6.** Total vehicle weight  $m_t = 1422$  kg, chassis weight  $m_1 = 1/5 m_t = 284.4$  kg and cab weight  $m_2 = 4/5 m_t = 1137.6$  kg.



**Figure 7.** Total vehicle weight  $m_t = 1422$  kg, chassis weight  $m_1 = 1/5 m_t = 284.4$  kg and cab weight  $m_2 = 4/5 m_t = 1137.6$  kg

For the case  $c_1 = -24275.56$ ,  $k_1 = 27548.62$ ,  $c_2 = 2225761.36$ ,  $k_2 = 60265.65$  we get the simulation result Fig. 7. The figure shows that the values of positions  $x_1$  and  $x_2$  are of infinite values. As a matter of fact, that deformation (dynamic crush) cannot be infinite, it is not possible to determine the time of the dynamic crush - a model unsuitable.

If we exchange the weight, that is, the weight of the chassis  $m_1 = 1137.6$  kg and the weight of the passenger compartment  $m_2 = 284.4$  kg we get the following results.

For the case  $c_1 = -3091.65$  Ns / m,  $k_1 = 8746.5$  N / m,  $c_2 = 6850074.77$  Ns / m,  $k_2 = 19581.12$  N/m we get the simulation result (see Fig. 8). In this figure, we can see that the dynamic crush of the chassis  $m_1$  is 0.75 m and is achieved in 0.08 s. For passenger compartment  $m_2$ , the dynamic crush is 0.75 m and is also achieved in 0.08 s (measured data show that the dynamic crush is 0.75 m and is achieved in 0.08 s (see Fig. 5)). This model can fully represent the vehicle crash scenario because this dynamic crush obtained from the simulation result (see Fig. 8) is very similar to the dynamic crush obtained from the measured, experimental data (see Fig. 5).

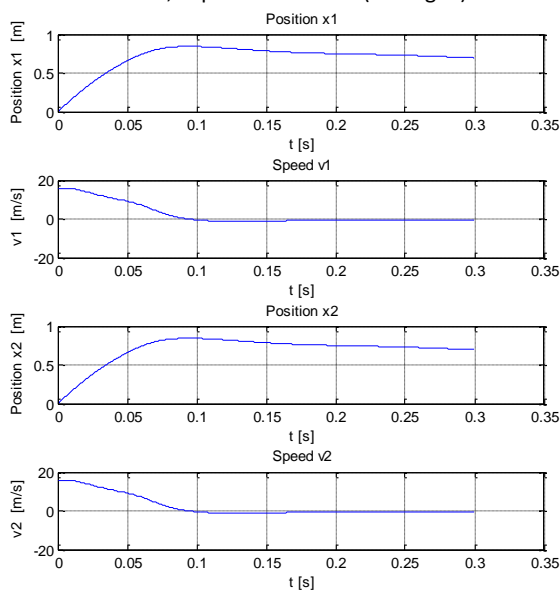


Figure 8. Total vehicle weight  $m_t = 1422$  kg, chassis weight  $m_1 = 4 / 5m_t = 1137.6$  kg and cab weight  $m_2 = 1 / 5m_t = 284.4$  kg

The course of the force  $F_{k2}$  in the spring  $k_2$  as a function of time can be seen in Fig. 9.

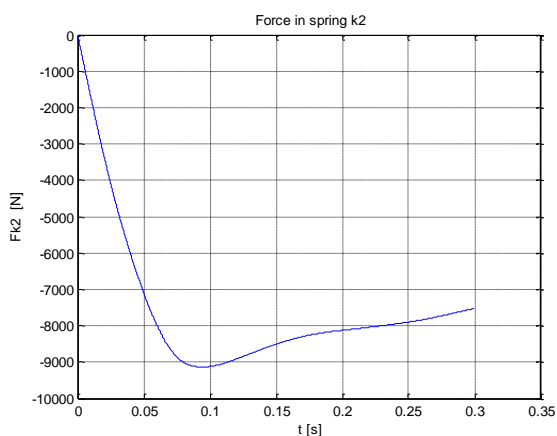


Figure 9. The force  $F_{k2}$  in the spring  $k_2$  as a function of time t

For the case of  $c_1 = 43732.53$  Ns / m,  $k_1 = -618.33$  N / m,  $c_2 = 97909.12$  Ns / m,  $k_2 = 1370014.95$  N/m we get the simulation result (see Fig. 10). In this figure, we can see that the dynamic crush of the chassis  $m_1$  is 0.3 m and is achieved in 0.04 s. For passenger compartment  $m_2$ , the dynamic crush is 0.4 m and is also achieved in 0.04 s (measured data show that the dynamic crush is 0.75 m and is achieved in 0.08 s (see Fig. 5)). This model does not fully represent the crash scenario.

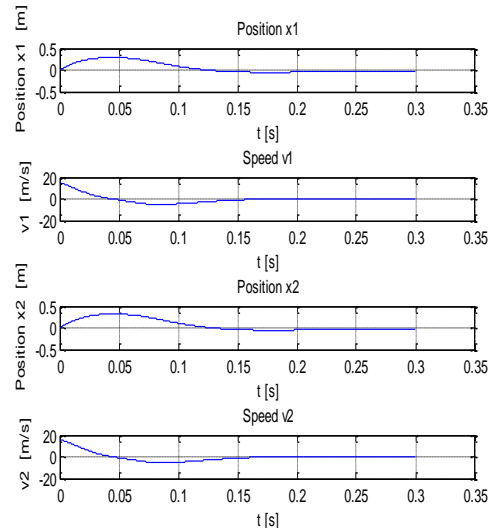


Figure 10. Total vehicle weight  $m_t = 1422$  kg, chassis weight  $m_1 = 4 / 5m_t = 1137.6$  kg and cab weight  $m_2 = 1 / 5m_t = 284.4$  kg

## 5 CONCLUSION

The results obtained and their discussion show that for the case where the mass of the chassis  $m_1 = 4/5 m_t$  and the mass of the passenger compartment  $m_2 = 1/5 m_t$ , where  $m_t = 1422$  kg (the mass of the chassis is greater than the mass of the passenger compartment) and the parameters thus obtained are:  $c_1 = -3091.65$  Ns / m,  $k_1 = 8746.5$  N / m,  $c_2 = 6850074.77$  Ns / m,  $k_2 = 19581.12$  N/m, the dynamic crush of the 2\_MSD model obtained in this way is closest to the dynamic crush which is determined from the experimental data (see Fig. 8, Fig. 5) and finally the dynamic crush is reached at the same time of 0.08 seconds for both cases, ie for 2\_MSD model and experimental output.

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