LOAD TORQUE ANALYSIS OF INDUCTION MACHINE

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This paper is focused on the load torque evaluation of the induction machine. There is described the problem of load torque oscillations and its impact to other electromagnetic variables in induction machine. In this article, the issue of modulated stator current is resolved and behaviors of amplitude and phase modulation are separated using demodulation algorithms after that. Newly, the variable load torque is retroactively determined from coefficients of amplitude and phase modulation not from nondemodulated stator current frequency spectrum. At last, theoretically acquired knowledge is compared with real measurement and different values of amplitude and phase modulation obtained from the mathematical motor model and from the real measurement are discussed. In the future, this analysis can be used as a basis for online diagnosis of driven load.

KEYWORDS
induction machine, diagnosis, amplitude modulation, phase modulation, stator current signature, load torque oscillation, spectral analysis.

NOMENCLATURE

IM – Induction Machine, **AM** – Amplitude Modulation, **PM** – Phase Modulation, **AD** – Amplitude Demodulation, **JAMP** – Joint Amplitude Phase Modulation, **MCDA** – Motor Current Demodulation Analysis, **MMFR** – Magnetomotive Force of Rotor, **DC** – Direct Current

1. INTRODUCTION

Since the 80s of last century, the issue of dealing with the diagnosis of electromechanical objects is very often discussed topic. The proof of the importance of electromechanical objects diagnosis is foundation of global symposium on diagnosing of electrical machinery under the auspices of the IEEE in 1997. This symposium is named: Symposium on Diagnostics for Electrical Machines Power Electric and Drives (SDEMPED). Every two years, scientists from around the world who deal with specifically the diagnosis of electric machines and drives meet and discuss new trends and future developments in this field of professional activity. [Jaksch 2009], [Jaksch 2010]

In many electrical systems, key element is motor. From the point of view of simple construction and high efficiency, the majority of all installed motors are induction machines today. For functional diagnosis and eventual continuous monitoring of IM, electrical quantities entering into the machine (stator current and voltage) and magnetic and mechanical quantities that are generated by the machine are available (speed, rotor position, stray magnetic flux, the shaft voltage, electromagnetic torque). Waveforms of these variables are strongly influenced by the actual motor condition or the condition of the connected load (unbalanced supply network, defects of winding or insulation, the state of bearings, dynamic and static eccentricity, angular and parallel misalignment, clutch failure and other). From time records of motor variables and their analysis, the current condition of the motor, its components, or even the status of the connected load can be continuously evaluated.

Waveforms of mechanical quantities such as vibration or noise can be recorded using accelerometers and sensors for noise levels sensing which are located close to or directly on the examined subject. Unfortunately, such sensors are often costly, and they can not be used in some applications. Diagnostics of IM using stator current analysis is therefore more comfortable. Actual condition of the motor is calculated from the stator currents, which may be obtained away from the subject of interest using e.g. the Rogowski coils placed around the motor power cables. However, these coils must be of sufficient precision. The time record of current is converted into the frequency domain and the amplitudes of the sidebands at different frequencies are compared with the first harmonic (usually 50 Hz in EU and 60 Hz in USA).

At the beginning of the twenty-first century, many studies dealing with the diagnosis directly detecting the state of IM has been published. It is mentioned in the literature [Casada 1995], [Welch 2015] that if the load torque of IM is torsionally affected, the stator current signature analysis as a tool for the diagnostic needs attains higher sensitivity e.g. compared to vibration analysis. For this reason, stator current spectral analysis is ideally suited for the evaluation of connected loads, which reflect variable load torque in a fault state. The aim of this work is to find the way how reliably evaluate oscillating load torque from stator currents only to be able to use this study for diagnosing faults, failures, and various factors manifested by load torque oscillations in systems that are driven by IM.

2. LOAD TORQUE OSCILLATIONS

At steady state, most electromechanical systems are operating with constant load torque which the induction machine has to equalize at all times. However, in some cases, the shaft of IM can be strained by variable load torque. For simplicity, variable load torque can be written as the sum of steady-state part load torque and oscillating component as:

$$\Gamma_z = \Gamma_0 + \Gamma_p \cdot \cos(\omega_p t) \qquad (1)$$

Where Γ_0 is average value of load torque, Γ_p denotes amplitude of variable load torque, $\omega_p = 2\pi f_p$ and f_p denotes frequency of oscillations. Due to the existence of electromechanical and electromagnetic linkages inside an induction motor, it can be proved that the variable load torque affects other electromagnetic quantities which the induction motor produces during operation. After applying the variable load torque on the shaft, rotor of induction motor starts to oscillate around a constant mechanical angular velocity which is proportional to the average load. Since the rotor magnetomotive force which is excited by current in the rotor is dependent on the actual rotor position, MMFR will be phase modulated and after transformation of the MMFR to stator coordinate system, the magnetomotive force can be expressed as:

$$f_{mr}(\vartheta, t) = F_{mr} \cdot cos(p_p\vartheta - \omega_s t - \beta \cdot cos(\omega_p t) - \varphi_{rs}) \qquad (2)$$

Where F_{mr} is amplitude of MMFR respecting machine geometry and the current in rotor circuit, p_p denotes motor pole pair number, ϑ denotes angle in the stator coordinate system (Fig. 2), $\omega_s = 2\pi f_s$.

fs is frequency of the supply network, φ_{rs} represents phase angle between the magnetic axes of the rotor and stator in electrical degrees, t is time and β denotes phase modulation factor. [Blödt 2006]

$$\beta = p_p \cdot \frac{\Gamma_p}{J \cdot {\omega_p}^2} \qquad (3)$$

J is moment of inertia. The phase modulated MMFR is reflected in the total magnetomotive force in machine air gap and the magnetomotive force ultimately deforms stator current of the induction machine which is powered from the supply network. If IM has to balance variable load torque, modulated stator current for the first phase can be written as:

$$\begin{aligned} i_f(t) = \{I_\mu - I_r cos[\alpha\, cos(\omega_p t) - \varphi_r]\}\, sin(\omega_s t) - \\ -I_r\, cos(\omega_s t)\, sin[\alpha\, cos(\omega_p t) - \varphi_r] \end{aligned} \qquad (4)$$

Where I_μ is amplitude of magnetizing current, I_r is amplitude of rotor current, modulation factor α is proportional to β and angle φ_r expresses the phase shift of the resulting magnetic field against the rotor magnetic field. The following flowchart (Fig. 1) summarizes all that is happening inside the motor when the induction machine is forced to balance variable load torque.

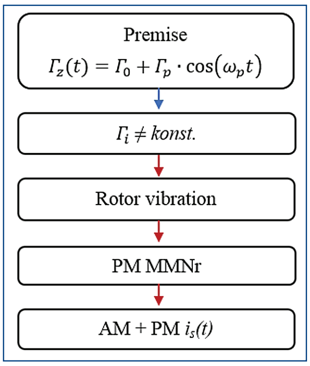

Figure 1. Diagram

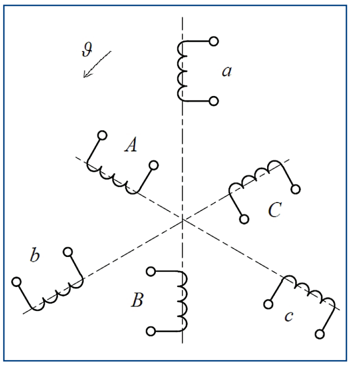

Figure 2. Induction machine

3. SPECTRAL ANALYSIS OF STATOR CURRENTS

Expression (4) contains components indicating the amplitude modulation and the phase modulation of the stator current. However, this relationship does not respect the moment of inertia of rotating masses due to which there is a phase delay between AM and PM. After modifying equation (4) it could also be seen that the variable load torque at a certain frequency – f_p is located in the stator current at frequencies $f_{1.HARM} \pm f_p$. This is a significant disadvantage for diagnostics of the rotor dynamics, because the frequencies which are obtained from the stator current spectrum are superimposed on the basic harmonic (50Hz) and their evaluation is indirect. And so e.g. the load torque oscillations at 5 Hz appear in the stator current at frequencies 45 Hz and 55 Hz. In addition, the amplitude of the left sideband in the current frequency spectrum has a different value than the right one. This is due to the phase delay between PM and AM and so in the frequency spectrum, the left sideband increases and the right sideband decreases. That's why the magnitudes of the side frequency bands say nothing about the "size" of load torque oscillations.

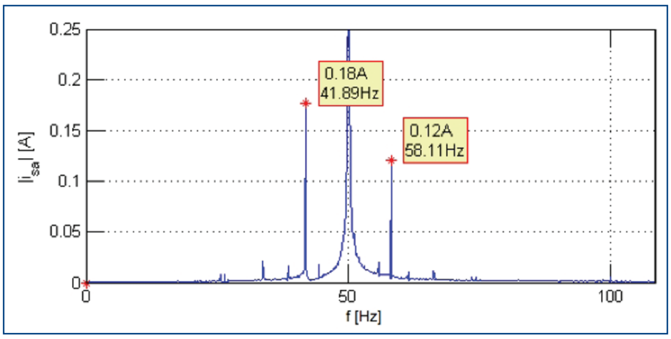

Figure 3. Detail of the frequency spectrum of stator current when Γ_z is not constant

In Fig. 3 is shown frequency spectrum and difference of the sideband sizes due to the phase delay between AM and PM is demonstrated. If the current waveforms would be modulated without the phase delay, vectors of AM and PM would be orthogonal relative to each other and the resulting amplitudes of the left and right frequency components would have reached the same size. Their magnitudes could be expressed using the following equation from the Pythagorean theorem: [Tuma 2009]

$$I_{L,P}=\sqrt{\left(\frac{\alpha_{AM}}{2}\right)^2+\left(\frac{\beta_{PM}}{2}\right)^2} \tag{5}$$

Where α_{AM} [A] is amplitude modulation factor and β_{PM} [A·rad] is the phase modulation factor.

If there is a phase delay between PM and AM by an angle φ, the vectors, expressing the effects of AM and PM, are not orthogonal and the amplitudes of side frequency bands in frequency spectrum can not be already obtained from the simple Pythagorean theorem, but from Cosine theorem:

$$I_{L,P}=\sqrt{\left(\frac{\alpha_{AM}}{2}\right)^2+\left(\frac{\beta_{PM}}{2}\right)^2+2\cdot\left(\frac{\alpha_{AM}}{2}\right)\cdot\left(\frac{\beta_{PM}}{2}\right)\cdot sin(\pm\varphi)} \tag{6}$$

All of these facts are represented graphically in Fig. 4. Relationships (5), (6) and units of their modulation factors respect equation (7).

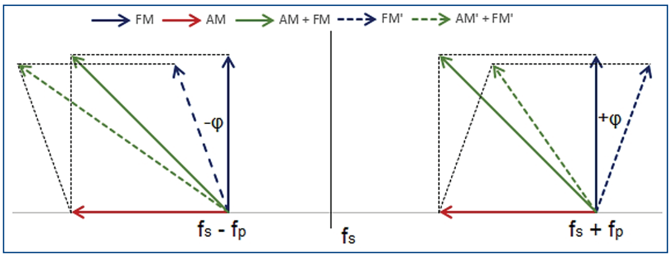

Figure 4. The graphical representation of the effect of moment of inertia on the side bands $I_{L,P}$

In the following analysis, the relationship (4) is necessary to replace. A substitution is performed using the general signal that is amplitude and phase modulated.

$$i_f(t)=(I_0+\alpha_{AM}\cdot sin\Omega t)\cdot sin\left(\omega t+\frac{\beta_{PM}}{I_0}\cdot sin\Omega t+\varphi_{AP}\right) \tag{7}$$

I_0 is amplitude of the stator current which corresponds to average load of the induction motor. The value can be obtained from load characteristics of the machine. Further on, the signal contains three unknowns: amplitude modulation factor – α_{AM}, phase modulation factor – β_{PM} and angle $\varphi_{AP}=\varphi$ which defines the phase delay between AM and PM, $\omega=\omega_s$ a $\Omega=\omega_p$.

3.1. Stator current amplitude and phase demodulation

Load torque oscillations are reflected in the stator current as Joint Amplitude Phase Modulation (JAPM), and thus the diagnosis from the frequency spectrum of nondemodulated stator current would not be very useful (shown in Fig. 3). It is advantages at the beginning to demodulate stator current waveform and to distinguish the influence of AM and PM. After that, to carry out evaluation of variable load torque from acquired modulation coefficients. For amplitude demodulation, Envelope Analysis is often used, when the information about a carrier signal is lost because of the demodulated process and the result is only amplitude and frequency of the modulating signal. For IM with negligible phase unbalance, the amplitude demodulation of stator current may be carried out using Eq. (8). But this relationship presupposes the possibility of measuring the current of all three phases.

$$I_{AD}=\sqrt{\frac{2}{3}(i_1(t)^2+i_2(t)^2+i_3(t)^2)} \tag{8}$$

The AD does not use complex mathematical operations, such as the Hilbert transform, therefore a demodulation process is faster, but its quality is dependent on the size of the stator phase unbalance. Entire AD analysis of stator current is based on the fact that at any moment the stator currents of all phases – $i_1(t)$, $i_2(t)$, $i_3(t)$ are monitored. After neglecting the noise in the supply network and accepting some simplifications, the stator currents are pure sine waves during the normal reliable operation of motor. These sinusoids are phase delayed by 120° relative to each other. In the summation of these three sine waves, the result would always be ideally zero. However, if the demodulation algorithm corresponding to the relationship (8) was applied to these three sine signals (the stator currents), we got a DC signal of constant amplitude – I_0.

In a situation when the stator currents are amplitude modulated and after its AD using Eq. (8), the constant DC component is obtained and on this one a oscillatory component is superimposed (Eq. (9)). The oscillatory component is proportional to variable load torque. After filtering the DC component and converting the signal into a frequency spectrum, the amplitude modulation factor – α_{AM} and frequency of load torque oscillations are possible to acquire.

The Fig. 6 shows the used algorithm graphically. In the left column there is a waveform of three currents of healthy motor model (without AM). Amplitudes of currents are 10 A and supply frequency is 1 Hz. In the right column there is the same record, but in this case the currents are AM with signal – $i_{osc} = sin(100\pi t)$. If the AD algorithm was applied on currents of the healthy motor model, it was achieved constant DC signal with amplitude equal to the maximum value of the phase stator currents. For motor, where stator currents were AM, the result had been the sum of the DC component and the amplitude modulated component – $i_{osc} = sin(100\pi t)$.

$$I_{AM} = I_0 + \alpha_{AM} \cdot sin\Omega t \qquad (9)$$

Similarly, it would have been even if the modulated signal contained more frequencies in itself. In Eq. (9) would appear the next summand which should have its own amplitude and frequency. Demodulation algorithm would work in the same way if the carrier signal had higher frequency than the modulation signal. This is more common for AM and PM in engineering practice. For diagnostics of rotor dynamic from the measurement of stator current, area of interest are below the first harmonic. This relates to the fact that each induction motor behaves as a low pass filter. That's why frequencies of variable load torque just about to 50 Hz will be examined.

Phase demodulation can be performed by mathematical calculation from spectral analysis of the AD signal and the nondemodulated frequency domain of stator phase current. It is not necessary to use for the phase demodulation any demodulation algorithm. By comparing the frequency specters of these two signals (frequency domains), a system of two quadratic equations with two variables is possible to create. By solving quadratic equations, we get always a two double roots – β_{PM} a φ_{AP}. After quantifying α_{AM}, β_{PM}, φ_{AP} and knowledge of the average load of induction motor, variable load torque supplied to the motor shaft can be reconstructed. The following flowchart (Fig. 5) captures the whole demodulation process.

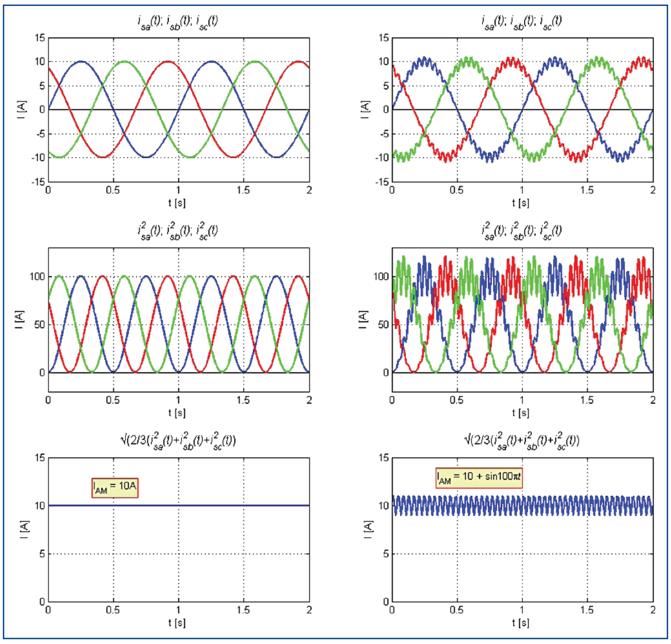

Figure 6. The graphical representation of the used algorithm for AD of stator currents

4. EVALUATION OF ROTOR DYNAMICS

The induction machine is multi parametric nonlinear dynamic system which is characterized by its transfer functions. For diagnosis of rotor dynamics associated with the load torque oscillations, it is advantageous firstly to determine relationship between the load torque and stator currents. The characteristic transfer function, which determines the amplitude and phase response of stator currents, can be obtained from the control theory. As written above, since each induction motor behaves as a low pass filter, the transfer function is also preliminary indication what frequency band it is possible to diagnose from the phase currents measurement. For a chosen motor, frequency band of interest was set up to 50 Hz. At this frequency, the damping of AM and PM already has been considerable, therefore it makes no sense to quantify further coefficients of modulations for the higher frequency. Modulations of currents from dynamic load on frequencies higher than 50 Hz would not be measurable in practice [Kroupa 2015].

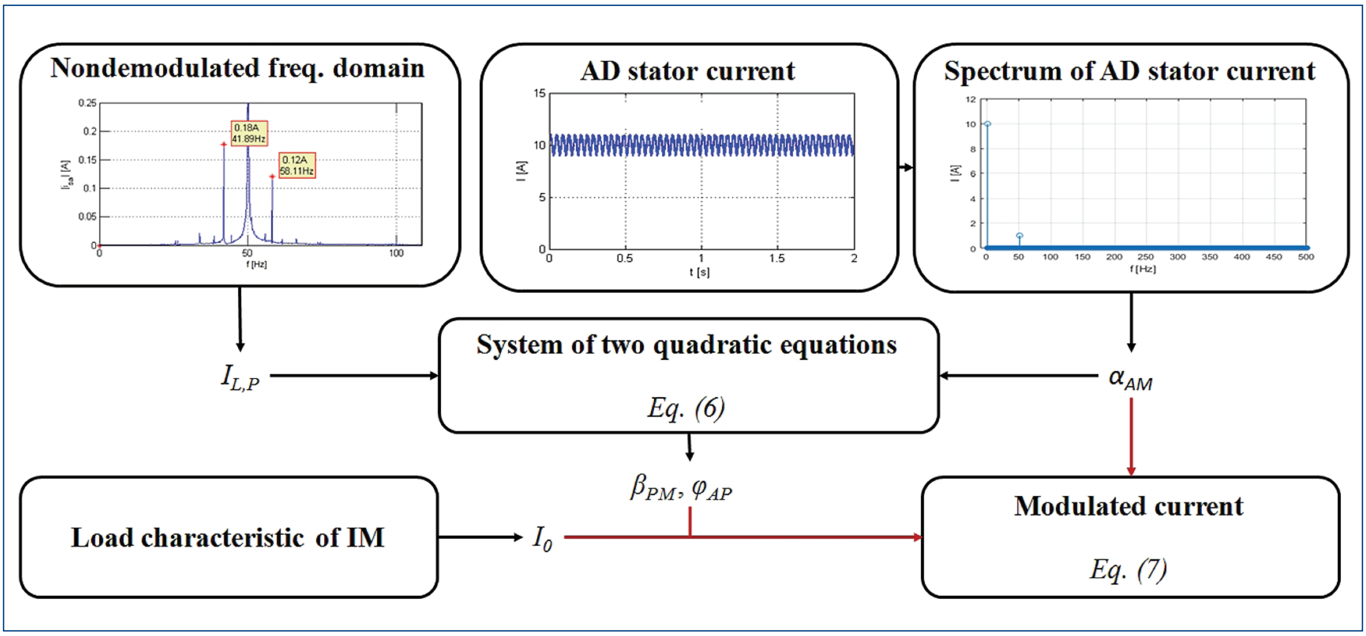

Figure 5. The demodulation process

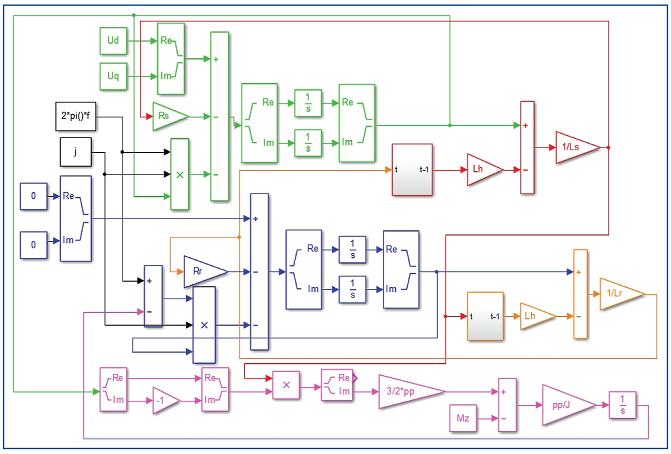

Figure 7. the mathematical model of induction machine

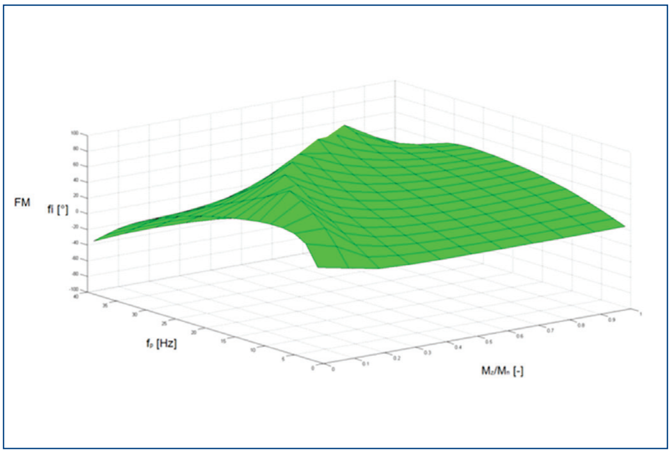

Figure 9. φ_{AP} between AM and PM of stator current when $\Gamma_z \neq$ konst.;$\Gamma_p = 0.01\ \Gamma_z$

Analytically derived relationships were tested in Matlab Simulink 2013. The model of induction machine based on the theory of general machine was compiled. (Fig. 7) To save computation time, the mathematical motor model was modeled in the coordinates "*d,q*" using Clark and Park transformations. The model consists of two voltage equations for the rotor and stator, two equations of flux linkages and equation of motion. A detailed description of the model can be found in the literature [Kroupa 2015] in Chapter 4.

Known parameters: P = 2,2 kW, R_s = 4,5 Ω, R_r'= 3,39 Ω, L_s= 0,0104H, L_r' = 0,0104 H, L_m = 0,256 H, p_p = 2, J = 0,005 kg/m^2, Γ_n = 15,1 Nm, n_n = 1400 min^{-1} were inserted in the model. Later the model was subjected to a series of simulations with variable torque oscillations corresponding with Eq. (1). The value of the oscillation amplitude was chosen as 1% of the rated load torque. At higher levels of the oscillations it is preserved linear nature of AM and PM. If dynamic load torque reached a higher level e.g. twice as large ones, the values of individual modulations of current could also rose twice. For actual quantification of modulation coefficients and phase delay, the mathematical model of IM was linearized. The linearization is based on literature [Alexandrovitz 1980] and it was carried out on 10 points of loads in the range from 0 to 100% of nominal torque.

The modulation coefficients were quantified and the surfaces of AM and PM were obtained. The surfaces on Fig. 8 and Fig. 9 are limited by two factors: the average load torque of IM and the frequency of dynamic torque. From these surfaces and from the knowledge of average load, the variable load torque supplied to the motor shaft can be determined. The AM and PM for tested motor can be seen in Fig. 8. The surfaces of modulations are unique for each IM. Their values depend on all parameters of a mathematical motor model. As can be seen from Fig. 8, the variable load torque is not always easy to identify from one of these surfaces because of their different character. While AM increases with average load of machine, PM decreases. And thus when IM is low loaded, AM of stator current is easier to measure than PM and conversely. At this point PM may not be readily measurable and evaluation of variable load torque might not be possible from PM only. Therefore, both the modulation is always important to know. With increasing frequency of oscillations, AM and PM respect the character of the low-pass filter. The surfaces achieve amplification on the cutoff frequency. The phase delay between AM and PM is visible in Fig. 9.

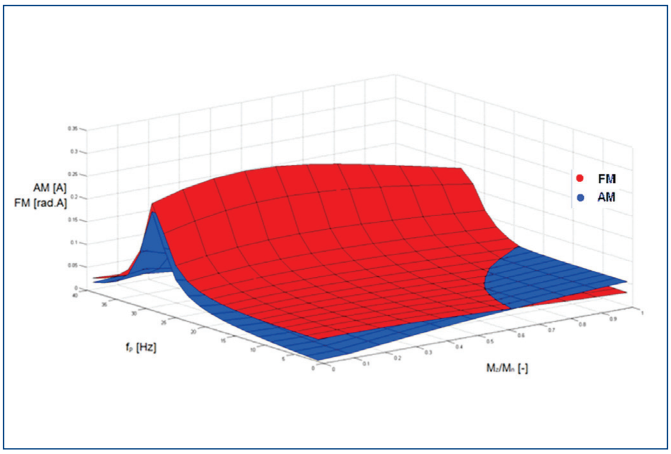

Figure 8. AM and PM of stator current when $\Gamma_z \neq$ konst.;$\Gamma_p = 0.01\ \Gamma_z$

5. MEASUREMENT RESULTS

The theoretical analysis was subsequently supported by the real measurement, when the real motor was loaded by variable load torque and modulation of all phase stator currents were studied. For practical reasons, the motor was loaded from half of its nominal torque, because it is not common practice that the motor would operate in areas of low load in the long term. The amplitude of load torque oscillations was chosen in a range up to 10 % of the rated motor torque and oscillation frequencies were set from 2 Hz to 12 Hz. Unfortunately, a driven load did not allow higher frequency oscillations. After determining of AM and PM factors, these coefficients were compared with the surfaces of AM and PM and relative deviations of the mathematical model against the real measurement were determined. Data from the mathematical motor model are regarded as the references. Some relative deviations can be seen in Tab. 1.

Load torque			Measured		Calculated		Rel. deviation	
Γ_z	Γ_p	f_p	α_{AM}	β_{PM}	α_{AM}	β_{FM}	δ_{AM}	δ_{PM}
Nm	Nm	Hz	A	rad	A	rad	%	%
8,55	0,66	6,00	0,13	0,28	0,151	0,184	15,77	34,91
8,57	0,82	8,00	0,17	0,32	0,187	0,236	9,88	25,68
9,42	0,44	4,00	0,10	0,11	0,115	0,109	15,20	2,46
9,40	0,46	6,00	0,10	0,11	0,113	0,123	13,20	10,01
11,03	0,77	6,00	0,20	0,25	0,213	0,191	6,70	24,57
11,02	0,71	10,00	0,18	0,23	0,196	0,192	6,63	17,27
11,76	0,44	8,00	0,12	0,14	0,127	0,109	2,74	24,03
12,57	0,61	6,00	0,18	0,20	0,185	0,169	2,78	14,25
12,55	0,39	8,00	0,11	0,12	0,118	0,095	7,27	19,18
12,55	0,31	10,00	0,09	0,10	0,094	0,085	0,86	15,79
14,08	0,33	4,00	0,11	0,09	0,108	0,077	1,82	12,90
14,12	0,32	6,00	0,10	0,09	0,105	0,073	4,60	16,97
14,09	0,32	8,00	0,10	0,10	0,104	0,081	4,30	15,09

Table 1 The relative deviation of AM and PM

6. CONCLUSION

The problems concerning variable load torque supplied to the motor shaft were discussed in this article. Influence of variable load torque in phase stator currents was mathematically derived and a way to evaluate variable load torque was found from phase stator currents only. The whole theory is supported by the real measurement. In Tab. 1, the relative deviations between actually measured data and data obtained from mathematical model are shown. These deviations achieved relatively significant values. This was due basically by three circumstances: inaccuracies of current sensors, not exactly determined moment of inertia of rotating masses (moment of inertia of load was not known, it was estimated) and motor parameters which were used in the mathematical motor model. The parameters were calculated for rated torque, that's why the relative deviation of AM and PM reach the lowest values of about 14 Nm. The parameters were determined by method reported in the literature [Patocka 2006]. In practice to achieve greater accuracy, the parameters of the motor model would have to be determined for the entire area of load and to create parametric mathematical motor model functionally dependent on medium load. In spite of these inaccuracies it can be said that the real measurement confirms the theoretical analysis and diagnosis of rotor dynamics associated with variable load torque is possible to calculate from stator current measurement with acceptable accuracy.

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