SW TOOLS FOR STRUCTURAL DESIGN OF PROFILES, CONSIDERING ITS STABILITY

TOMAS KALINA, MARTIN KRATOCHVIL, FILIP HRDLICKA, IVANA MAZINOVA

Faculty of Mechanical Engineering,

University of West Bohemia, Pilsen, Czech Republic

When designing lattice structures, or even structures in general, standard profiles are often used. These are profiles of I, U, T, L, SHS, RHS, CHS, EHS shapes, etc. All the basic formulas for stiffness and strength calculations are known for standard used profile shapes. However, in established practice there are no approaches for simply optimization or for evaluating the stability of profiles. This paper deals with a detailed analysis of two SW tools for structural design of profiles, considering its stability and enabling shape optimization for the most efficient use of the material. Both previously mentioned tools are based on the theory of prof. Ashby but each of them is handled in a slightly different way. The first tool works with purely analytical formulas and enables profile evaluation and manual optimization. The second tool is based on analytical formulas, but uses numerical solvers and allows full profile optimization.

KEYWORDS

Buckling, Cross-section shape, Material efficiency, Materials selection optimization, Numerical optimization, Shape factor.

1 INTRODUCTION

In both mechanical engineering and civil engineering, structures (e.g. lattice construction) are often designed from standard profiles. All the basic formulas for stiffness and strength calculations are known for standard used profile shapes (I, U, T, L, SHS, RHS, CHS, EHS etc.). For these calculations there is a large number of calculators.. However, in established practice there are no approaches for optimization or for evaluating the stability of profiles (now let's omit FEM analysis, topology optimization, topography optimization and other types of FEM optimizations).
Where :

SHS = Square Hollow Section, RHS = Rectangular Hollow Sections
CHS = Circular Hollow Section, EHS = Elliptical Hollow Section

This paper deals with a detailed analysis of two SW tools for structural design of profiles, considering its stability and enabling shape optimization for the most efficient use of the material.
Both mentioned tools are **based on the theory of prof. Ashby** [Ashby 2011], [Ashby 2013] but each is handled in a slightly different way. The first tool is created in the MS Excel 2016 environment, works with purely analytical formulas and enables profile evaluation and **manual optimization**. The second tool is based on analytical formulas but uses numerical solvers and also allows **full profile optimization** according to specified requirements.
Another example of the application of prof. Ashby's theory is given in paper [Florian 2017]. This paper deals with connection of prof. Ashby's theory with optimization interface of software ANSYS. This link allows solving complicated multi-criterial problems that cannot be solved analytically.

2 INTRODUCTION TO THEORY (PROF. ASHBY'S)

Mechanical and civil structures often consist of basic (standardized) profiles, which are, for example, **ties/bars** loaded by tension, **shafts** loaded by torsion, **columns** loaded by pressure, **beams** loaded to bend, whether beams **one-end-fixed** or **on two supports**, etc. Suitable shapes according to the way of their load are generally known for these standard applications.
In the case of **ties/bars loaded** by **simple tension** (in the cross-sectional axis), it essentially only depends on the cross-sectional area of the profile. Whether it is a round or square rod, a tube, or an open profile has no effect on stiffness or strength. However, a different situation already occurs with other load methods. In the case of **columns loaded by compression**, the cross-sectional area of the profile is also essential for their stiffness and strength, however, it must be checked here whether a loss of stability due to buckling can occur. In this case, the well-known Euler equation for the critical force applies (1).

$$F_C = \frac{n^2 \, \pi^2 \, E \, I_{min}}{L^2} \quad [N] \tag{1}$$

Where: $\underline{F_c}$ is a critical force [N], $\underline{n}$ is a constant that depends on the end-constraints [-]; $\underline{E}$ is a Young's modulus of the material of the section [GPa]; $\underline{I_{min}}$ is a smallest second moment of the area [m⁴], $\underline{L}$ is a column length [m].

Euler equation for the critical force (1) takes into account the minimum second moment of area of cross-section with respect to the length of the beam but this may not always be sufficient to assess the stability of thin-walled hollow sections, as it is possible to design a beam that will have a large second moment of area of cross-section but have a very thin, unstable wall (see below). Furthermore, Tetmajer's theory which defines buckling in the plasticity, is often used in technical practice. In the case of steel structures, we are well below the limit of elasticity, so this theory is not discussed further here.
When **loaded with a pure torsion** it is usually advantageous to use circular hollow sections. Circular hollow sections are rotationally symmetrical about their axis, but mainly have a very good ratio between the cross-sectional area and the torsional sectional modulus.

cross section shape	$A \, [m^2]$	$Q \, [m^3]$	
round bar	$\dfrac{\pi D^2}{4}$	$\dfrac{\pi D^3}{16}$	(2;3)
CHS = Circular Hollow Section	$\dfrac{\pi (D^2 - d^2)}{4}$	$\dfrac{\pi (D^4 - d^4)}{16}$	(4;5)

Where: $\underline{A}$ - Cross-sectional area of the section [m²], $\underline{Q}$ - Torsional section modulus [m³], $\underline{D}$ - large diameter [m], $\underline{d}$ - small diam. [m]

The farther the material is from the profile axis, the more it contributes to the increase of the torsional section modulus (towards the cross-sectional area (2) and (4)), which results from the mentioned formulas (3) and (5). That means that a better ratio of cross-section area to the torsional section modulus is achieved by using a large-diameter tube and a thin wall. In theory, we could be approaching the extreme with a pipe of infinite diameter with infinitesimal wall thickness. However, such a pipe (e.g. an aluminium foil pipe) would be highly unstable as the slightest other load would result in loss of stability and collapse of the pipe wall. A breach of stability would be caused by the sheer weight of the pipe in such extreme cases.
A similar situation occurs with profiles **loaded by bending**. More precisely, the further the material is from the axis of the profile

perpendicular to the load force, the more it contributes to the expansion of the second moment of area of the section (*I* [m⁴]). I-shaped profiles or RHS profiles are appropriate for these applications. The stands that connect the material above and below the neutral axis are necessary to hold the **"carrier material"** in the correct position, and to ensure the stability of the profile, but contribute at least to increasing the second moment of area (Fig. 1).
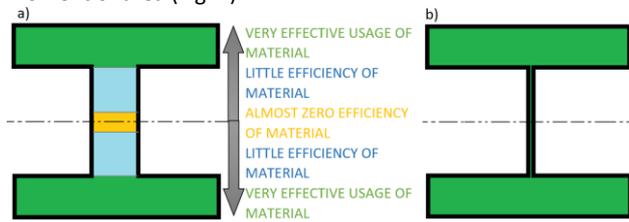
Figure 1. a) A description of the efficiency of material use on the real I-profile, b) "ideal" I-profile for the most efficient use of the material, without taking stability into account

cross section shape	A $[m^2]$	I $[m^4]$	Z $[m^3]$	
square bar	$B_0{}^2$	$\dfrac{B_0{}^4}{12}$	$\dfrac{B_0{}^3}{6}$	(6;7;8)
rectangular bar	BH	$\dfrac{BH^3}{12}$	$\dfrac{BH^2}{6}$	(9;10;11)
RHS	$BH - bh$	$\dfrac{BH^3 - bh^3}{12}$	$\dfrac{BH^3 - bh^3}{6\,H}$	(12;13;14)
EHS	$\dfrac{\pi}{4}(BH - bh)$	$\dfrac{\pi(BH^3 - bh^3)}{64}$	$\dfrac{\pi(BH^3 - bh^3)}{32\,H}$	(15;16;17)

Where: *A* - Cross-sectional area of the section [m²], *I* - Second moment of area of the section [m⁴], *Z* - Section modulus of the section [m³], *B₀* - edge length for the square reference section [m], *H* - external height [m], *B* - external width [m], *h* - internal height [m], *b* - internal width [m]

Thus, in the case of profiles loaded with torsion and bending, it is **generally known which profile shapes are suitable for the load** (Fig. 2), and how changes in dimensions for a given shape affect changes in second moment of area values (6) to (17) and stability.

However, there are **no general rules for finding optimal boundaries between these parameters**, i.e. designing a profile to use the material **as efficiently as possible** (i.e. using as little material as possible) while ensuring structural stability.
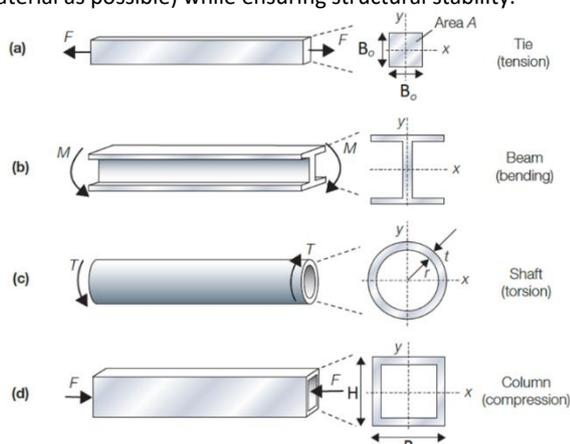
Figure 2. Common modes of loading and the section shapes that are chosen to support them [Ashby 2011, pp.247]

This problem is **solved by prof. Ashby's theory** [ASHBY 2011], which introduces the so-called **Shape factor**, a dimensionless quantity that takes into account shape efficiency. The form factor for elastic bend (18) is **defined as the ratio of the bending stiffness of the profile considered (S)** to **the bending stiffness for the square reference section (S₀)**, which is considered to be a square bar with side width b₀.

$$\phi_B^e = \frac{S}{S_0} \ [-] \tag{18}$$

Where: ϕ_B^e - Macro shape factor for elastic bending deflection [-], *S* - Bending stiffness [N/m], *S₀* - Bending stiffness for the square reference section [N/m]

If we put in (18) [Ashby 2011, pp.248] the formula for bending stiffness (*S* [N/m]) (19) and after *Iₒ*, which is the second moment of area for the square reference section, we substitute a spaced formula (20). And if we consider comparing beams from the same material, we get an adjusted formula (21). This newly obtained formula is generally valid for comparing all different beam shapes from the same material. We can also see from the formula (21) that the **shape factor does not depend on the absolute size**, but only on the ratio of the second moment of area, which is visualized on Fig. 3. The shape factor de facto indicates the number of times the cut beam is stiffer against a square bar of the same cross-sectional area (square reference section).

$$S = \frac{E\,I}{L^2} \ \left[\frac{N}{m}\right] \tag{19}$$

$$I_O = \frac{b_0{}^4}{12} = \frac{A^2}{12} \ [m^4] \tag{20}$$

$$\phi_B^e = \frac{S}{S_0} = \frac{E\,I}{E\,I_0} = \frac{12\,I}{A^2} [-] \tag{21}$$

Where: ϕ_B^e - Macro shape factor for elastic bending deflection [-], *S* - Bending stiffness [N/m], *S₀* - Bending stiffness for the square reference section [N/m], *Iₒ* - Second moment of area of the square reference section [m⁴]
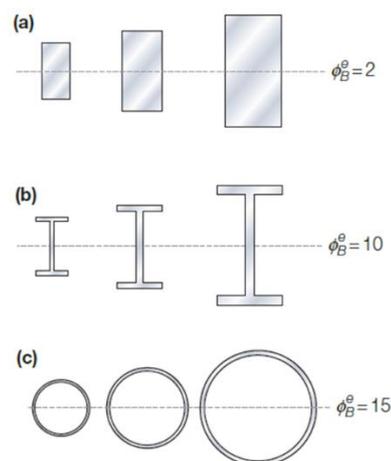
Figure 3. A set of rectangular sections, I-sections and tubes in which members of a set differ in size but not in shape [Ashby 2011, pp.251]

The Shape factor for onset of plasticity or failure in bending is based on the ratios of the **Section modulus of the section** (*Z* and *Z₀* [m³]) and the resulting formula is expressed as (22) [ASHBY 2011, pp.254].

$$\phi_B^f = \frac{Z}{Z_0} = \frac{6\,Z}{A^{\frac{3}{2}}} \ [-] \tag{22}$$

Where: ϕ_B^f *- Macro shape factor for onset of plasticity or failure in bending [-],* **$\underline{Z}$** *- Section modulus of the section [m³],* **$\underline{Z_0}$** *- Section modulus of the square reference section [m³]*

Similarly, the above applies to the **shape factor for elastic torsional deflection**, where the final formula (23) is obtained. See more - theory of prof. Ashby [ASHBY 2011, pp.251].

$$\phi_T^e = \frac{S_T}{S_{T0}} = \frac{K}{K_0} = 7.14\frac{K}{A^2} \ [-] \tag{23}$$

Where: ϕ_T^e *- Macro shape factor for elastic torsional deflection [-],* **$\underline{K}$** *- Torsional moment of area [m⁴],* **$\underline{K_0}$** *- Torsional moment of area for the square reference section [m³],* **$\underline{S_T}$** *- Torsional stiffness [N.m],* **$\underline{S_{T0}}$** *- Torsional stiffness for the square reference section [N.m]*

Furthermore, as the formula above the formula for the **shape factor for onset of plasticity or failure in torsion** (24) is given. See more - theory of prof. Ashby [ASHBY 2011, pp.256].

$$\phi_T^f = \frac{Q}{Q_0} = 4.8\frac{Q}{A^{\frac{3}{2}}} \ [-] \tag{24}$$

Where: ϕ_T^f *- Macro shape factor for onset of plasticity or failure in torsion [-],* **$\underline{Q}$** *- Torsional section modulus [m³],* **$\underline{Q_0}$** *- Torsional section modulus for the square reference section [m³]*

According to the **theory of prof. Ashby**, the **upper limits of the Shape Factors** for the most commonly used materials were further determined, which are listed in Tab. 1. These limits have been set with regard to production technology and functionality. E.g. I-profile with the same dimensions as, for example, steel is not produced from wood. Although we would be able to produce a thin-walled profile from wood, such a profile will not be usable in practice. Thus, we point to the reason why in practice steel structures are made of different shaped profiles of the type: I, U, L, etc., but for wooden structures a typical crossbeam a beam with a rectangular cross-section (loaded in the direction perpendicular to the shorter side of the rectangle) and not e.g. I - profile or hollow sections.

material	$(\phi_B^e)_{MAX}$	$(\phi_T^e)_{MAX}$	$\left(\phi_B^f\right)_{MAX}$	$\left(\phi_T^f\right)_{MAX}$
structural steel	65	25	13	7
EN AW 6061	44	31	10	8
GFRP/CFRP	39	26	9	7
Polymers	12	8	5	4
Woods	5	1	3	1
elastomers	<6	3	-	-

Table 1. Empirical upper limits for the shape factors – entire table [-] [ASHBY 2011, pp.258]

However, the **unique idea of this whole theory** (which does not occur in other theories) is that limit values can be set for individual shape factors, which guarantee that compliance with them will not result in a limitation due to loss of stability (so-called Limits imposed by local buckling).
Formulas (25) and (26) [ASHBY 2011, pp.260] are empirically determined formulas, which indicate the limits that should not be exceeded for a given material, otherwise there is a risk of loss

of beam stability, although according to analytical calculations will comply.

$$(\phi_B^e)_{MAX} \approx 2.3 \left(\frac{E}{\sigma_f}\right)^{\frac{1}{2}} \ [-] \tag{25}$$

$$\left(\phi_B^f\right)_{MAX} \approx \sqrt{(\phi_B^e)_{MAX}} \ [-] \tag{26}$$

Where: $\underline{\sigma_f}$ *- Yield or failure strength of the material of the section [MPa],* $(\phi_B^e)_{MAX}$ *and* $\left(\phi_B^f\right)_{MAX}$ *[-] - Upper limits on shape efficiency.*

3 TOOL NO. 1 - CALCULATOR IN MS EXCEL

3.1 Basic information and principles

This SW tool is used to easily determine the appropriate combination of cross-sectional shape, dimensions, and beam material for specific selected boundary conditions. The main benefit of this SW tool over common calculators is that the stability of the profiles is also taken into account and it is possible to compare several materials at once. Taking into account the stability of the profiles, a simple analytical solution of the beam on the strut is not meant here. **Based on the Shape factors**, it is verified whether the proposed beams have suitable dimensions and a sufficiently thick wall relative to their basic dimensions, so that the beam wall cannot collapse (lose stability) under bending or torsional loading, but the beams are not oversized. In addition to conventional formulas for stiffness and strength, the **calculator is largely based on the theory of prof. Ashby** [Ashby 2011], [Ashby 2013]. The SW tool is created in MS Excel 2016.

The calculator can currently be used for **beams on two supports**, or **one-end-fixed beams** that are loaded by **bending** or **torsion**. The calculator works with two basic shapes of profile cross-section, namely a **RHS** = rectangular hollow section (a special case of which is a **SHS** = square hollow section) and an **EHS** = elliptical hollow section (of which a **CHS** = circular hollow section is a special case).

When using this SW tool, it is possible to aim for the lowest possible weight, fulfilment of the required deflection or torsion, always in compliance with other specified conditions, such as the minimum required safety against yield strength (or against tensile (ultimate) strength) and maximum permissible dimensions. Using the tool, the user performs de facto **manual optimization** of parameters, where he has immediate feedback on the effect of the change on the results.

3.2 Description of the organ structure of the SW tool

The basis of this SW tool is a table for entering material parameters of the considered materials (Fig.4). and table for defining input boundary conditions (Fig.5). To table (Fig.4) it is necessary to enter the required material parameters. In the table (Fig.5) it is necessary to select the boundary conditions of the calculation, such as: method of loading, type of beam, loading force, limit dimensions, etc.

Another part of the calculator are the works-tables (Fig.6), the number of which is equal to the multiple of the number of compared materials and the number of cross-sectional shapes, and the **user performs his own manual optimization** here. The principle of function, resp. optimization will be explained below.

The last part is the results table (Fig.7), which summarizes the results for all compared combinations of shapes and materials, where the user can quickly evaluate the output data and compare them with each other.

3.3 Description of the function structure of the SW tool

The user can choose the default materials or can define his own material in the table with material parameters (Fig.4). As mentioned above, when designing a beam loaded by bending, it is necessary to enter the parameters: density (ρ [kg/m³]), Young's modulus of the material (E [GPa]), yield strength (σ_y [MPa]) and/or tensile (ultimate) strength (σ_{ts} [MPa]), and in the case of torsional calculation also the shear modulus (G [GPa]). Based on the entered values, according to (29) and (31), the limit shape factors for buckling prevention are calculated.

Comparable materials - Properties							
mark	ρ [kg/m3]	E [GPa]	G [GPa]	σ ts [MPa]	σ y [MPa]	$\varphi Be\ max$ [-]	$\varphi Bf\ max$ [-]
material 1 HSS	7850	210	80	800	600	37.3	6.10
material 2 S235	7850	210	80	360	250	55.6	7.45
material 3 AW6060-T6	2690	71	26	210	160	42.3	6.50
material 4 SRP	1220	8.3		207		14.6	3.82

Figure 4. Material input table

Furthermore, in the table with boundary conditions (Fig.5) the user can select (from the drop-down menu) the method of loading (bending / torsion) and the type of beam (on two supports or one-end-fixed) and enter numerical values manually: loading force F_x [N], maximum permissible dimensions X_{max} [mm] a Y_{max} [mm], total length L [mm] and maximum permissible weight m_{max} [kg].

Other optional parameters are: minimum required safety against yield strength, max. permissible deflection (in case of bending), or max. permissible twist (in case of torsion). When the user fills in these optional data, it can help him, for example, when comparing or evaluating the results, because the SW takes into account compliance with or non-compliance with the specified limits by conditional formatting.

Max. profile parameters		
method of loading	bending	-
boundary condition	on 2 supports	-
loading force Fx	1500	N
dimension x	100	mm
dimension y	100	mm
total length L	1000	mm
mass m	20.0	kg
optional:		
request. min. safety s σy	1.5	-
request. max. defl. w	2.5	mm
request. max. twist		
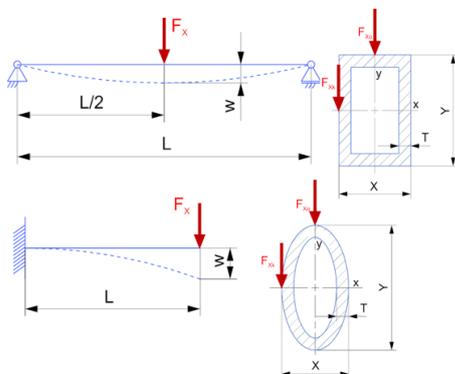
Figure 5. Input table of profile parameters and load method

After entering both input tables, it is possible to move to the **work tables** (Fig.6). Work table for each material and shape combination consists of two parts. The left part is always fully automatically calculated from the specified boundary conditions and serves as a kind of guide in which areas the user can move approximately.

The values given in the left part are always based on the input maximum permissible dimensions X_{max} a Y_{max} and for these dimensions, the upper and lower limits of the profile wall thickness are further calculated..

The maximum possible wall thickness is determined on the basis of the maximum dimensions X_{max} a Y_{max} and maximum permissible weight m_{max} *(if with dimensions X_{max} a Y_{max} has been exceeded t_{max} [mm], thus the maximum permissible weight will be exceeded!)*. Formulas for t_{max} are derived in (27) and (28). To distinguish the formulas used designation: RHS = Rectangular Hollow Sections and EHS = Elliptical Hollow Section.

$$t_{\max\ (EHS)} = \frac{1}{4} \cdot \left[(x + y) - \sqrt{(x+y)^2 - \left(\frac{16}{\pi} \cdot A_{max}\right)} \right] \ [m] \quad (27)$$

$$t_{\max\ (RHS)} = \frac{1}{4} \cdot \left[(x + y) - \sqrt{(x+y)^2 - (4 \cdot A_{max})} \right] \ [m] \quad (28)$$

Where: A_{max} is the maximum possible cross - sectional area [m²] (based on the maximum permissible weight and length of the beam).

When adding the parameters for **EHS** to (22) and we know formula for upper limit of shape factor (26), we get formula for shape factor $\Phi_B{}^f{}_{(EHS)}$ [-] (29). Using $\Phi_B{}^f{}_{(EHS)}$ and $\Phi_B{}^f{}_{max}$ [-] we can express the minimum possible wall thickness $t_{min(EHS)}$ (30), which guarantees us the stability of the profile (if t_{min} [-] is exceeded with these dimensions X_{max} and Y_{max}, then there will be a risk of loss of profile stability).

$$\phi^f_{B\ (EHS)} = \frac{3}{2\sqrt{\pi}} \cdot \sqrt{\frac{\frac{y}{2}}{t}} \cdot \frac{1+3\frac{x}{y}}{\left(1+\frac{x}{y}\right)^{\frac{3}{2}}} \ \leq \phi^f_{B\ MAX} \ [-] \quad (29)$$

$$\rightarrow\ t_{\min\ (EHS)} = \frac{9\ y^2\ (y+3x)^2}{8\pi \left(\phi^f_{B\ MAX}\right)^2 \cdot (x+y)^3} \ [m] \quad (30)$$

The same procedure can be applied to a **RHS** and from formulas (31) we can express the minimum possible wall thickness $t_{min\ (RHS)}$ (32).

$$\phi^f_{B\ (RHS)} = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{y}{t}} \cdot \frac{1+3\frac{x}{y}}{\left(1+\frac{x}{y}\right)^{\frac{3}{2}}} \ \leq \phi^f_{B\ MAX} \ [-] \quad (31)$$

$$\rightarrow\ t_{min\ (RHS)} = \frac{y^2\ (y+3x)^2}{2 \cdot \left(\phi^f_{B\ MAX}\right)^2 \cdot (x+y)^3} \ [m] \quad (32)$$

The remaining values in the relevant column are already calculated for the specified wall thickness according to the analytical formulas. For example, formulas for the second moment of area of the section for elliptical (33) and rectangular (34) hollow sections are given:

$$I_{(EHS)} = \frac{\pi}{64} \cdot \{x \cdot y^3 - [(x - 2t) \cdot (y - 2t)^3]\} \ [m^4] \quad (33)$$

$$I_{(RHS)} = \frac{1}{12} \cdot \{x \cdot y^3 - [(x - 2t) \cdot (y - 2t)^3]\} \ [m^4] \quad (34)$$

From the formulas for second moments of area (33) and (34) it is then possible to express the section modulus of the section (Z) (35) and (36):

$$Z_{(EHS)} = \frac{\pi}{32y} \cdot \{x \cdot y^3 - [(x - 2t) \cdot (y - 2t)^3]\} \; [m^4] \qquad (35)$$

$$Z_{(RHS)} = \frac{1}{6y} \cdot \{x \cdot y^3 - [(x - 2t) \cdot (y - 2t)^3]\} \; [m^4] \qquad (36)$$

The values in the left part of the table depend purely on the two input tables, and do not change as part of the manual optimization performed in the right part.

In the right part of the table, the user then performs his own **manual profile optimization**. The user has the option to correct the X_{corr} [mm], Y_{corr} [mm] and t_{corr} [mm] values in this part of table. For the X_{corr} and Y_{corr} parameters, the user has the option to select any values that do not exceed the input values X_{max} and Y_{max}.

According to the selected values of the X_{corr} and Y_{corr} parameters, the upper and lower limits of the t_{corr} parameter are recalculated.

The limit values t_{corr} are determined in the same way as in the left table, with the only difference that it is based on the corrected values X_{corr} and Y_{corr} and not on the limit values X_{max} and Y_{max}.

That is, these are the limit values (lower and upper) of the t_{corr} parameter, according to the corrected values of the X_{corr} and Y_{corr} parameters.

After entering the values the user has the option to evaluate the entered profile according to the values in the outputs section. Here they are calculated for the selected profile and given boundary conditions: safety against yield strength (only if the yield strength is not specified, the safety against tensile ultimate strength is evaluated), safety against both shape factors, real deflection or twist and real weight.

At the bottom of the table are then the values of the parameters from which the above outputs are calculated. These values are not necessary for the evaluation itself, but they can be used, for example, in comparison, or to visualize what has an effect on the final results.

Note: *It is common for shape factors that safety values (especially for the shape factors for onset of plasticity or failure) do not reach high values. This is not a yield strength safety equivalent and should not be considered as such. That is, it is not appropriate to require high safety, as can be the case with safety against yield strength. A value of 1.1 can easily be a sufficient safety factor for the shape factor $\Phi_B{}^f$.*

After entering all the values, the user has the opportunity to evaluate the outputs described above and, based on this, repeatedly change the values of X_{corr}, Y_{corr} and t_{corr} until he reaches some sub-optimal solution.

The precondition for such a sub-optimal solution is that all the mentioned safety features (possibly also deflection and twisting) approach the set limits to the limit and the lowest possible weight is achieved.

Figure 6. Calculated limit values on the left side, corrected on the right side

This manual profile optimization procedure should be repeated for all material and shape combinations.

After performing all optimizations, it is possible to proceed to the table of results (Fig.7), where all important compared outputs are summarized, i.e. dimensions, safety, deflection, or twist and weight. Based on the summarized results, the user evaluates and selects the resulting beam parameters. When evaluating, it is appropriate, in addition to the best result, to evaluate the price of the material and the price of the profile production technology.

Note 1*: This means that if the profile of aluminium or PEEK is 1% better than the steel structure, which will be several times cheaper, it is appropriate to take this into account.*

Note 2*: If it is a product made of plastic, or e.g. foam, machined or cast product, the user can choose any size. When it comes to designing a structure from standardized profiles made of aluminium alloy or steel, it is, of course, appropriate to take into account the range sold when choosing the dimensions.*

	Resulting parameters								
	elliptical / circular tube				square / rectangular tube				
	HSS	S235	AW6060-T6	SRP	HSS	S235	AW6060-T6	SRP	
x corr	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	mm
y corr	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	mm
t corr	5.00	5.00	5.00	5.00	3.00	3.00	3.00	3.00	mm
s σ y	54.02	22.51	14.41	-	58.47	24.36	15.59	-	-
s σ ts	67.73	30.48	17.78	18.64	69.66	31.35	18.29	20.17	-
s φBe max	3.90	5.82	4.43	1.53	2.24	3.33	2.54	0.87	-
s φBf max	1.61	1.97	1.72	1.01	1.06	1.29	1.13	0.66	-
deflection w max twist	0.088	0.088	0.261	2.230	0.081	0.081	0.241	2.061	mm
m real	11.714	11.714	4.014	1.821	9.137	9.137	3.131	1.420	kg

Figure 7. The resulting table with overviews of all materials and shapes

4 TOOL NO. 2 - NUMERIC OPTIMIZER ("TOY")

4.1 Introduction of the program

The TOY program is a tutorial program designed to practice material selection and beam profile optimization. The program is focused on the theory of assessing the material performance and the shape of the profile created by prof. Ashby. The aim of the program is to simplify the calculations of the values of material indicators, shape factors or their combinations. Along with these results, the program provides information on weight, amount of reduced stress or displacement. In addition to a simple check calculation, the program offers advanced functions intended to find the optimal dimensions for a given profile shape or to design dimensions and shapes for general input.

The program was created to implement advanced features for teaching profile optimization. This functionality is very important, because in the bachelor's program, in addition to simple calculation, it is necessary to teach the principles of structural optimization. Therefore, it is necessary to provide students a powerful tool that will allow them to work with the parameters of the structure and at the same time provide information about the useful properties of the proposed structure.

4.2 Initial assumption

The choice of a platform for creating a tutorial program in an academic environment is subject to specific circumstances. The problem is fixed limit of financial resources (unlike human resources), which leads to solution based on non-commercial basis. The following criteria were considered for the creation of the TOY program:

- development environment free of restrictive license terms
  - important for distribution in the academic environment
- cost-effectiveness for the university. The program and its distribution is funded by a grant (financial resources that cannot be increased)
  - developer costs
  - program distribution costs
  - the cost of a license to enable students to use the program
- possibility to run on the user's PC without restrictions
- support for various operating systems (Windows, Android, MacOS, Linux)
- support for various hardware platforms (x86-64, ARM, Apple Silicon, future platforms)
- easy creation of custom scripts
- scripting for automation of internal processes in the program
- ability to work with a higher degree of abstraction, making the code easier to maintain
- built-in advanced functions - linear algebra, solver of non-linear equations and function minimum finder. It is essential for the implementation of something like 'find the best solution'.

Due to the above requirements, an open-source solution was chosen. This eliminates the cost of acquiring a development environment and licensing to users. After considering several variants, it was decided to develop the application using the Python programming language and freely available libraries (Numpy, Scipy, Tkinter).

It seems appropriate to use an online platform, however, it is necessary to analyse the implementation and provide resources for running and administering the web server.

4.3 Description of the organ structure of the TOY
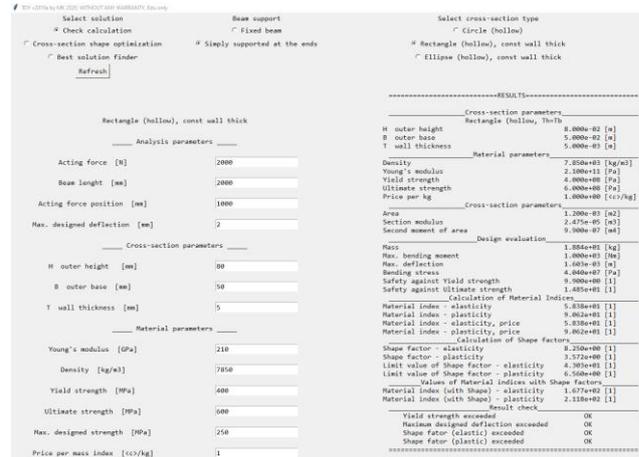
Figure 8. TOY layout

The TOY program consists of two parts, a graphical user interface and a task solver. The graphical interface layout is arranged in a grid by Tkinter library. The user interface consists of radio buttons, which can be used to select the type of task (check calculation, best solution for specified aspect ratios), beam support types (simply supported or fixed) and beam cross-section shape.
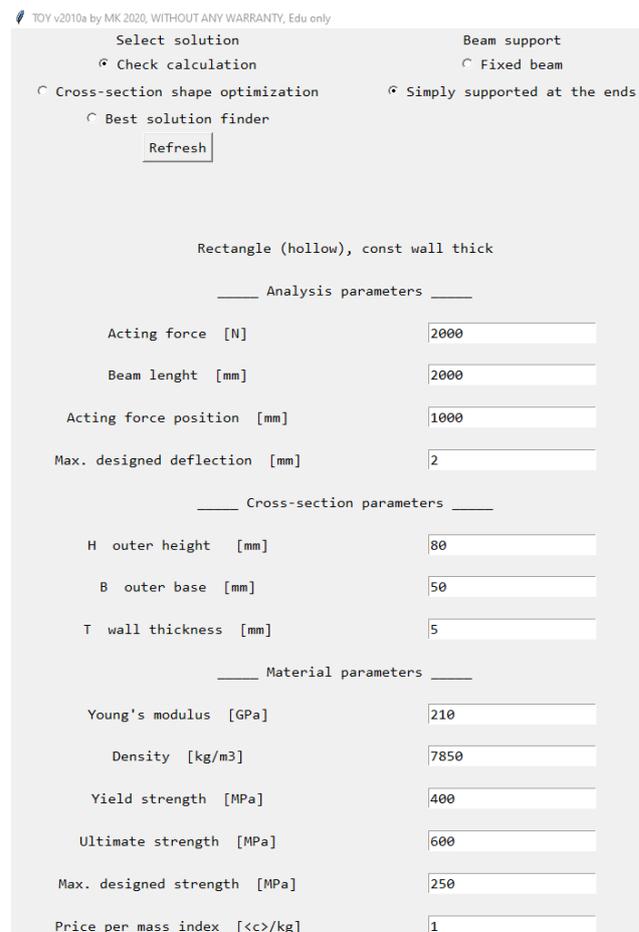
Figure 9. TOY solution parameter input window

The kernel is used to solve a user-specified task. It consists of a set of memory fields for input data and procedures that perform calculations and control the flow of data between memory cells. TOY uses a single unified kernel to solve all types of tasks.

```
            Select cross-section type
                ○ Circle (hollow)
         ⦿ Rectangle (hollow), const wall thick
           ○ Ellipse (hollow), const wall thick



========================RESULTS========================

_____Cross-section parameters_____
                  Rectangle (hollow, Th=Tb)
H  outer height                      8.000e-02 [m]
B  outer base                        5.000e-02 [m]
T  wall thickness                    5.000e-03 [m]
_____Material parameters_____
Density                              7.850e+03 [kg/m3]
Young's modulus                      2.100e+11 [Pa]
Yield strength                       4.000e+08 [Pa]
Ultimate strength                    6.000e+08 [Pa]
Price per kg                         1.000e+00 [<c>/kg]
_____Cross-section parameters_____
Area                                 1.200e-03 [m2]
Section modulus                      2.475e-05 [m3]
Second moment of area                9.900e-07 [m4]
_____Design evaluation_____
Mass                                 1.884e+01 [kg]
Max. bending moment                  1.000e+03 [Nm]
Max. deflection                      1.603e-03 [m]
Bending stress                       4.040e+07 [Pa]
Safety against Yield strength        9.900e+00 [1]
Safety against Ultimate strength     1.485e+01 [1]
_____Calculation of Material Indices_____
Material index - elasticity          5.838e+01 [1]
Material index - plasticity          9.062e+01 [1]
Material index - elasticity, price   5.838e+01 [1]
Material index - plasticity, price   9.062e+01 [1]
_____Calculation of Shape factors_____
Shape factor - elasticity            8.250e+00 [1]
Shape factor - plasticity            3.572e+00 [1]
Limit value of Shape factor - elasticity  4.303e+01 [1]
Limit value of Shape factor - plasticity  6.560e+00 [1]
_____Values of Material indices with Shape factors_____
Material index (with Shape) - elasticity  1.677e+02 [1]
Material index (with Shape) - plasticity  2.118e+02 [1]
_____Result check_____
    Yield strength exceeded              OK
    Maximum designed deflection exceeded OK
    Shape fator (elastic) exceeded       OK
    Shape fator (plastic) exceeded       OK
========================================================
```

Figure 10. TOY result window

4.4 Description of organ structure of the TOY
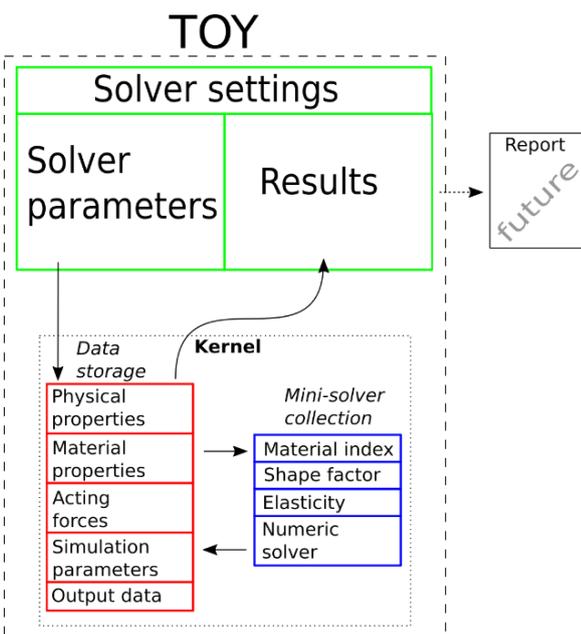
Figure 11. TOY block diagram

The graphical user interface was created using the Tkinter library. The introductory window allows the user to select the type of job, the type of beam support and the shape of the cross section. After the user selects from the options offered by the home screen, he presses the Refresh button. The window is then redrawn and enlarged, the task option is located in the new window on the top of the window below the bar. The left half of the window is used to enter the parameters of the task. Job parameters are entered in text fields. When all required fields are filled in, the user can press the Enter key. Pressing the Enter key activates the computer kernel.

After entering the command to solve the task, the computing kernel starts. It first copies the data from the GUI text boxes to its memory. In memory, data is divided according to its meaning. It then starts processing data using predefined procedures. Procedures are very simple functions (calculation of cross-sectional area and quadratic moment, maximum displacement, etc.), each procedure performs only a specific activity. Each procedure has the ability to run a different procedure. There are 3 levels of procedures used in the kernel. At the lowest level, individual calculation tasks are performed, higher layers take care of the continuity of the procedure (calculation of cross-sectional characteristics → stress, deflection → output processing). The top layer is a macro that determines how the program should work. The memory also includes variables that determine the type of job, beam placement, and cross-sectional shape.

The kernel was designed to be able to solve different types of tasks in one configuration. Simple procedures on the lowest layer can be freely run on higher layers on demand. Thanks to this, new functions can be added to the program relatively quickly.

The occurrence of duplicates in case if each type of process was performed by different code is limited alongside with it.

The kernel can handle 3 types of tasks:

   1. Control calculation, where material indicators and shape factors are calculated for beam parameters specified by user.

   2. Optimization of cross-section shape. In this case, the user enters the aspect ratios of cross-section dimensions, the program recommends optimal dimensions according user defined aspect ratios.

   3. Optimal solution. The user enters the limits of cross-section dimension parameters, the program tries to design the optimal dimensions and shape within these limits.

At the end of the calculation, a procedure is run to perform a basic (incomplete) test of the results. If the results are considered to be correct, the kernel converts the results to a readable form and sends them back to the GUI.

The output texts of the program are fully parameterized for the possibility of teaching foreign students. The parameters are arranged as a two-dimensional language-text field. This enables any number of languages to be added into the program. The program fully supports UTF-8. The description can be supplemented with other languages as well as any characters.

The TOY program is designed to be expanded with new features. Unlike previously mentioned Excel program, the TOY uses more generalized mathematical models. An example of this approach is a calculation of beam deflection. Both programs calculate beam deflection using Euler-Bernoulli beam theory. Excel program uses a formula, which is analytically derived from Euler-Bernoulli differential equation. This formula allows to calculate of the exact value, the parameters (support type, number of forces) are immutable. On the contrary, the TOY solves Euler-

Bernoulli equation as numeric integral. Thanks to numeral solution, there are arrays of loads in TOY's memory, so it's easy to modify TOY's solver for any possible variant (statically indeterminate beam, asymmetrically distributed load etc.). Disadvantage resulting from the numerical solution is that the result comes with a small deviation from analytic solution given by numeric (finite element) solver.

5 SAMPLE APPLICATION EXAMPLE

Both programs were compared according to the same assignment (Tab. 2, Tab. 3, Fig. 12 and Fig. 14). Assignment of this **example** was beam of **rectangular shape on two supports** loaded by force in the middle. **Steel S235JR** was chosen as a material. The safety factor (against Yield strength) was set to 2.5. The goal is to find the lightest beam.

Example parameters	Description
F = 3 000 N	Acting force
l = 1 500 mm	Length of beam
a = 750 mm	Acting force position
H = 100 mm	Max. height of beam
B = 100 mm	Max. width of beam
w = 10 mm	Max. deflection of beam
s = 2.5	Safety factor against Yield strength
s ($\phi_{B\ max}^{f}$) = 1	Safety factor against buckling

Table 2. Assignment of example

Material parameters	Description
E = 210 GPa	Young's modulus
ρ = 7 850 kg/m³	Density
σ_{ts} = 360 MPa	Ultimate strength
σ_y = 250 MPa	Yield strength

Table 3. Parameters of steel S235JR

5.1 Program Excel

Figure 12. The Excel assignment

The Excel program works on process trial and error method where finding the best solution takes user approximately about 5 minutes.

For entering values the best solution seems to be the profile with width 41 mm, height 100 mm and wall thickness 1.61 mm. The mass of this profile is 5.224 kg and the deflection 1.785 mm. All conditions have been met (Figure 13).

Figure 13. Results of excel

5.2 Program TOY

Figure 14. The TOY assignment

Figure 15. Results of TOY

Program TOY makes automatic design of profile based on those entered values. The calculation takes several seconds.

For entering values the best result seems to be the profile with width 45.21 mm, height 98.45 mm and wall thickness 1.577 mm. The mass of this profile is 5.219 kg and the deflection 1.786 mm (Figure 15).

corrected user values		
x corr	45.2	mm
y corr	98.5	mm
t corr ` = 1.15 <	1.577 <	3.98 mm
s σ y	2.54	-
s φBe max	1.59	-
s φBf max	0.99	-
deflection w max	1.787	mm
m real	5.218	kg
A `	443.2	mm2
Ix	562 024	mm4
W(B)x	11 417	mm3
σ(bend)	99	Mpa
φBe	34.9	[-]
φBf	7.54	[-]

Figure 16. Comparison of results from TOY in excel program

If we check results from the TOY program in the Excel (Figure 16) program we can find that the TOY program did not fulfil stability (ϕ_B^f) however the result is very close. And the reason is that the TOY program works based on numerical solving whereas the Excel works based on analytical solving.

Using a numerical solver, a large number of variants of profile dimensions were calculated. Figure 17 shows all the investigated solutions graphically (different ratios of outer width and height of the profile).

A dark area in the Figure 17 highlights a set of suitable solutions. The darker solution the smaller weight of a beam satisfying conditions of stress, deflection and stability. The closer the solution is to the black area the better. The dark area shows the set of best ratios of width and height of profile for assignment from chapter 4.

With a numerical solution we cannot say with certainty that we have found the optimal solution (global minimum of the function). There is usually only one optimal solution and it lies somewhere in the black area. However, all solutions from the dark area can be considered good solutions (sub-optimal).

In the Figure 17 we can also see the results of both our programs. Both of our solutions are located close to each other and both are in the middle of the dark zone.
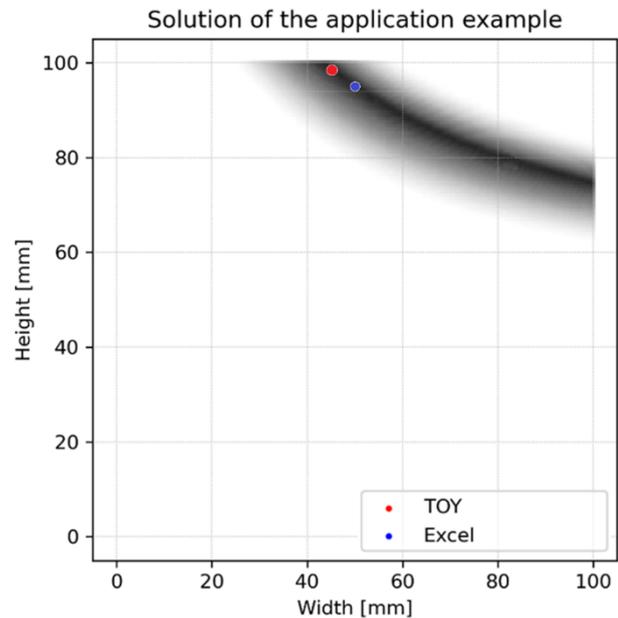
Figure 17. Representation of solutions

For the example, the data for the profile width 100 mm and height 75 mm (the solution from the other side of the dark side.) were calculated. The results for this profile are shown in Figure 18. All safety factors and deflection are met. The resulting weight is slightly greater than the results obtained with TOY and EXCEL.

corrected user values		
x corr	100.0	mm
y corr	75.0	mm
t corr = 0.95 <	1.330 <	1.67 mm
s σ y	2.64	-
s φBe max	2.15	-
s φBf max	1.00	-
deflection w max	2.258	mm
m real	5.398	kg

Figure 18. Results of profile 100x75x1.33

6 DISCUSSION AND EVALUATION OF SW TOOLS AND RESULTS

From the previous chapters it is obvious that we have two programs working on different principles with almost the same result.

In both programs a user can choose what kind of shape, profile, material, support and load (bending or torsion) he wants to calculate.

The advantage of the excel program is that the user can compare 4 different materials of two different shapes at one moment. Next one is that the excel program calculates results numerically and the user has a possibility to see how a change of entered

values affects results. The disadvantage of the excel program is a long time of calculating, in comparison with TOY program.

In the previous chapter there was mentioned that the TOY program did not fulfil stability (ϕ_B^f) and that is caused by the numerical solving method. This small inaccuracy could be perceived as a disadvantage but the purpose of both programs is to show to the user the best possible option. It is neccessary to find a real profile and to check it.

7   CONCLUSIONS

It is better to use TOY program when the user is looking for a quick tool for design. On the other hand the Excel program can be used as a great tool for educational purposes because the user can see all the steps of calculation process. So it can be easily understood how values correspond with one another and how is it influencing those final results.
Despite the differences between the programs, both of them are better to use than classic methods because they calculate moreover with stability of the profiles.

FINAL NOTE

The presented SW tools were created only for educational purposes. The authors of the SW tools are not responsible for errors or for faulty constructions created on the basis of the above calculations. The authors recommend to additionally verify the proposed structure before any real application.
Larger number of students will put the software tools to the test in the near future.
SW tools are free to download at:
www.home.zcu.cz/~mazini/

CONTACTS:

Ing. Filip Hrdlicka
Ing. Tomas Kalina
Ing. Martin Kratochvil
Ing. Ivana Mazinova, Ph. D.
University of West Bohemia, Faculty of Mechanical Engineering,
Univerzitni 8, Plzen, 306 14, Czech Republic
cficeriov@kks.zcu.cz , tkalina@kks.zcu.cz, kratochv@kks.zcu.cz, mazini@kks.zcu.cz

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