COMPARISON OF STANDARD AND NONSTANDARD GEAR ROOT FILLET CONSIDERING ROOT STRESS AND MANUFACTURING POSSIBILITIES

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In this paper, root stress, manufacturing possibilities and limitations of standard and nonstandard tooth root fillet of spur gears are examined. Nowadays, standard trochoidal root fillet is used the most as an economical trade-off between tooth strength and manufacturing complexity. A disadvantage of the trochoidal root fillet, especially in spur gears with less than 17 teeth, is undercutting which minimises strength of tooth root, introduces stress concentration, and deteriorates meshing conditions. This study focuses on different root designs, namely elliptical and novel cycloidal root fillets. Root stress in gears with different root designs is analysed by FEM and compared with analytical equations according to ISO. The possibility of producing nonstandard root fillet gears by hobbing and limitations of surface finishing are considered as well. Analysis reveals stress differences between each of the root fillet shapes and their relative advantages and disadvantages.

KEYWORDS

spur gear, root stress, tooth strength, elliptical curve, cycloidal curve, undercutting.

1 INTRODUCTION

Motion transmission gears fatigue life is affected by many factors. Two main approaches for gears fatigue life determination are based on contact stress and bending stress analysis. While contact strength of tooth surface is an important factor affecting transmission accuracy, heat and noise generation, and smooth run in general, on the other hand bending strength is crucial for torque transmission itself, since tooth breakage consequences are fatal.

Many authors have investigated possible ways of tooth bending strength improvement, and there is plenty of research materials available. Kapelevich and Shekhtman [Kapelevich 2009] have introduced one-sided involute asymmetry in spur gear to increase load carrying capacity. Hebbal, Math and Sheeparamatti [Hebbal 2009] investigated root fillet stress reduction by introducing stress relief features of different shapes (elliptical and circular holes) and sizes. Ingole et al. [Ingol 2015] created cylindrical holes in asymmetric teeth and observed the behaviour of root fillet stress drop depending on the shape and size of the hole. Ali and Sharma [Ali 2017] studied the combined effect of cylindrical and elliptical holes. Abu-Hamdeh and Alharthy [Abu-Hamdeh 2014] used one and multiple combinations of cylindrical holes for the same purpose. Pedersen [Pedersen 2010] and Zhao [Zhao 2014] tried to optimise tooltip shape and thus root fillet to increase loading capacity. Dong et al. [Dong 2022] and Uelpenich [Uelpenich 2019] focused on calculation times reduction dealing with the same task. Hebbal and Sowjanya [Sowjanya 2017] investigated reducing root stress in undercut gear by reducing the size of the gearbox. Zou et al. [Zou 2014] optimised root fillet using a cubic spline. Costopoulos and Spitas [Costopoulos 2005] found out that a circular root fillet compared to trochoidal root fillet is beneficial for gears with a small number of teeth from a root stress point of view. Sankar, Raj, and Nataraj [Sankar 2010] investigated circular fillets as well. Peng, Luo, and Ma [Peng 2019] designed an elliptical root fillet and compared its strength with the strength of standard root design using the method according to [ISO 6336-3] and [AGMA 2101].

In this paper, a wider range of standard and nonstandard root fillets are examined using multiple comparative methods. It is aimed at root shapes of simple mathematical descriptions, namely ellipsis and cycloid. Root stresses in the elliptical root, the cycloidal root, and two forms of trochoidal roots (full radius, non-full radius) are compared according to [ISO 6336-3] and FEM. The paper is organized as follows: Chapter 2 presents basic equations and the design of gear with different types of roots. In Chapter 3 stress analysis of each case is conducted and evaluated. Chapter 4 discusses manufacturing possibilities and limitations regarding hobbing and grinding. The last chapter concludes the research and evaluates outcomes.

2 DESIGN GEOMETRY OF GEARS

The first step is to calculate the coordinates of involute points according to equations 1 and 2 and trochoidal root points according to equations 3 and 4 [Nemcek 2003]. After that, these points are imported to CAD modeller, in which the teeth profiles are designed. The basic parameters of examined gears are shown in Tab. 1.

$$x = r \cdot \sin\varphi + g \cdot \cos(\varphi - \alpha_t) \tag{1}$$

$$y = r \cdot \cos\varphi + g \cdot \sin(\varphi - \alpha_t) \tag{2}$$

$$x = r \cdot \sin\varphi + g \cdot \cos(\psi - \varphi) \tag{3}$$

$$y = r \cdot \cos\varphi + g \cdot \sin(\psi - \varphi) \tag{4}$$

Parameters	symbol	value	unit
Number of teeth	z	13	[-]
Normal module	m_n	1	[-]
Pressure angle	α_n	20	[°]
Helix angle	β	0	[°]
Face width	b	10	[mm]

Table 1. - Parameters of examined gears

The elliptical and cycloidal roots are constructed using CAD modeller's embedded functions. These roots are connected to original involute, both are tangent to the involute in its point of beginning. Both nonstandard roots pass through the tooth space axis and root diameter intersection, and at this point are also tangent to the root diameter. Another constrain is that the main axis of ellipse is identical with the tooth space axis. As for the cycloid, the position of its base line is determined by previous constrains. Constructions of these roots are shown on Fig. 1 and Fig. 2.

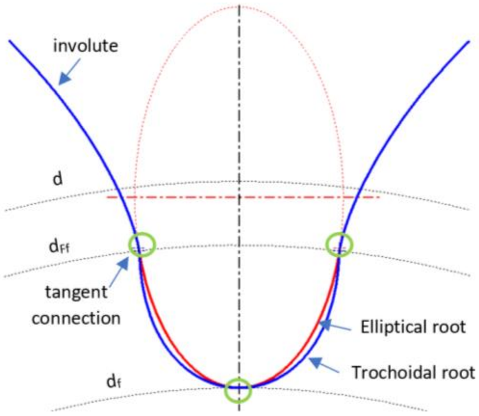

Figure 1. Construction of elliptical and trochoidal roots

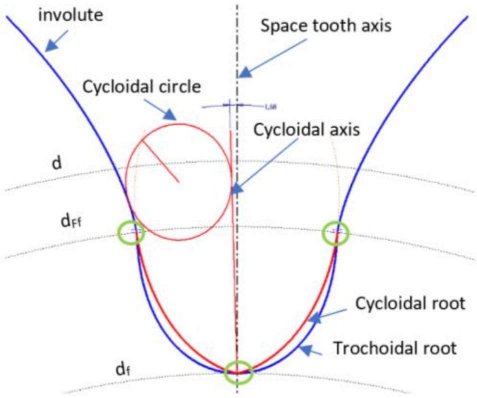

Figure 2. Construction of cycloidal and trochoidal roots

3 STRESS ANALYSIS

In this chapter, the stress analysis of standard and non-standard root fillet is performed. Root stress is calculated by finite element method (FEM) and analytical equations according to ISO standard. Results are compared and benefits of non-standard root fillets are evaluated. It's important to realize that purpose of all calculations (analytical and FEM) is only comparative and absolute values should not be taken as actual results. Actual results are relative changes of root stress values.

3.1 FEM root stress analysis

The gear analysis is performed for gear with parameters according to Tab.1. All analysed gears have the same basic parameters as already mentioned in previous chapter. They differ only in the shape of root fillet. For root stress analysis these roots are selected and compared:

a) Root fillet produced by standard hob

- Trochoidal – rounded root

- Trochoidal – non rounded root ($\rho_{fP}^{*} = 0{,}38$)

b) Root fillet produced by non-standard hob

- Elliptical

- Cycloidal

Boundary conditions and loading force

All gear models are transferred to FEM software. Boundary conditions are chosen as shown in Fig.3. Gears are loaded by modal force F = 500 N distributed across the entire face width of tooth edge represented by point A, which lies at tip circle d_a. In calculation, only the middle tooth is loaded. Loading modal force is tangent to base pitch circle d_b of the gear as shown in Fig. 4.

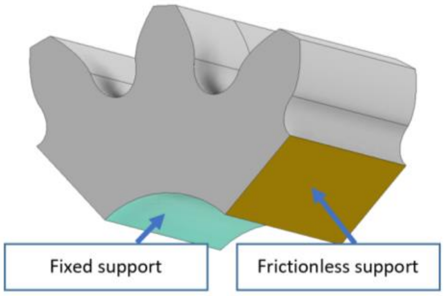

Figure 3. Boundary conditions

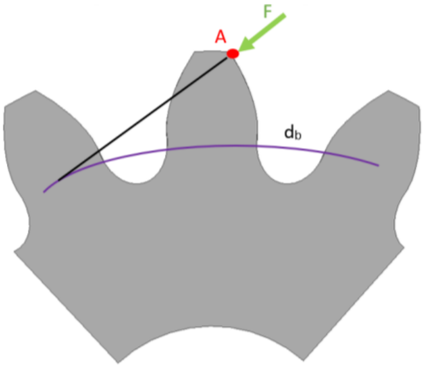

Figure 4. Loading force location

Root stress

Root stress is analysed in a plane in the middle of face width, in order to reduce influence of sharp edges at gear's side planes, as shown on Fig. 5. Also stress distribution along the tooth profile length (involute and root fillet) is shown on Fig. 6.

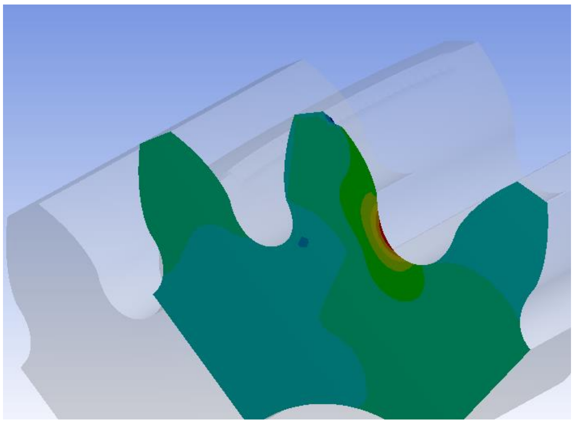

Figure 5. Root stress distribution in middle plane

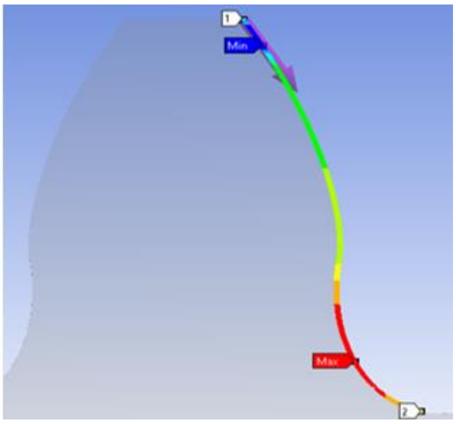

Figure 6. Root stress along the tooth profile length

Results of FEM root stress analysis are presented in Tab. 2. Data show that non-standard roots (elliptical, cycloidal) decrease value of root stress, compared to standard trochoidal roots.

Type of root	Max. root stress [MPa]	Max. root stress location at profile length [mm]
Trochoidal (non-rounded)	210,11	2,24
Trochoidal (rounded)	200,44	2,13
Elliptical	185,18	2,22
Cycloidal	161,41	2,20

Table 2. - FEM stress results

3.2 *Analytical root stress calculation*

In previous chapter, root stress was calculated by FEM. For comparison, another method for maximal root stress determination is used, namely analytical calculation according to ISO 6336:3. The most common mathematical model to calculate the maximal tooth root stress is 30-degree incline tangent method [5]. The critical point where the root stress has the highest value is defined by line which is inclined 30 degrees from tooth axis and is tangent to root fillet curve. Critical thickness S_{Fn} is defined as a distance between two critical points. Vertical distance h_{Fe} is defined between critical points connecting line and stress loading point on the tooth axis as shown in Fig. 7.

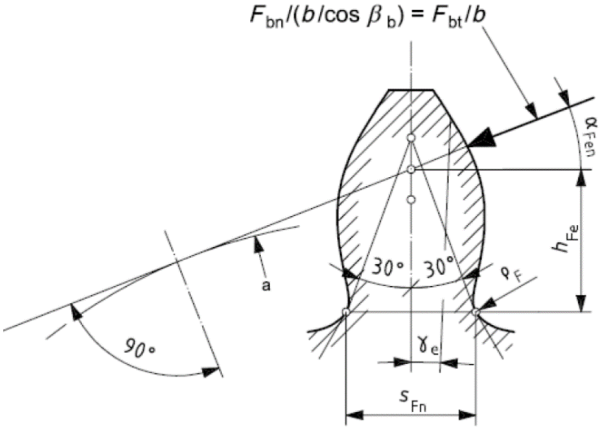

Figure 7. Mathematical model [ISO 6336-3]

Every type of root has a different radius of curvature in critical point, which has an influence on root stress calculation. ISO standard deals only with standard trochoidal fillets, curvature of other shapes must be measured in CAD modeller. The maximum of root stress can be expressed as

$$\sigma_F = \sigma_{F0} \cdot K_A \cdot K_V \cdot K_{F\beta} \cdot K_{F\alpha} \tag{5}$$

For comparison reasons the value of all K factors is considered 1. value of tooth root stress can be expressed as

$$\sigma_{F0} = \frac{F_t}{b \cdot m_n} \cdot Y_F \cdot Y_S \cdot Y_\beta \cdot Y_B \cdot Y_{DT}. \tag{6}$$

Where F_t is the nominal tangential load at the reference cylinder

b facewidth

m_n normal module.

All Y factors are calculated according to [ISO 6336-3]. Description of all analytical calculations is not in scope of this article. For more details refer to [ISO 6336-3].

The nominal Results of analytical calculations are shown in Tab. 3.

Type of root	Max. root stress [MPa]
Trochoidal (non-rounded)	228
Trochoidal (rounded)	216,45
Elliptical	199,66
Cycloidal	172,18

Table 3. - Analytical stress results

3.3 *Stress results*

The maximum of root stress was calculated using different methods (FEM, ISO). It was found out that using non-standard roots, especially in gear with lesser number of teeth, can be useful for reducing or completely removing undercutting, and improving tooth strength. Root stress in the elliptical root is reduced by 13,5% compared to trochoidal non-rounded root and 8,3% compared to trochoidal rounded root according to FEM results. Root stress in the cycloidal root is reduced by 30% compared to trochoidal non-rounded root and 24% compared to trochoidal rounded root according to FEM results. The rest of results, including analytical calculation results are shown in Tab.4.

	Improvement over nr. trochoid [%]		Improvement over r. trochoid [%]	
Type of root	FEM	ISO model	FEM	ISO model
Trochoidal (non-rounded)	-	-	-	-
Trochoidal (rounded)	4,8	5,3	-	-
Elliptical	13,5	14,2	13,5	14,2
Cycloidal	30,2	32,4	30,2	32,4

Table 4. - Stress comparative results

4 MANUFACTURING POSSIBILITIES AND LIMITATIONS

As proved above, non-standard fillets significantly improve tooth strength and indirectly affect other meshing properties. However, they also have certain limitations. For commercial utilization of non-standard root fillet, it is important that they could be produced by conventional methods.

4.1 *Hobbing process*

The most economical method of gears production is hobbing. Gears with standard trochoidal root and lesser number of teeth (generally <17) tends to suffer from undercutting. A tooth is undercut as consequence of hobbing tool movement as shown in Fig. 8. Using non-standard root reduces undercutting. That is caused by less material removal from root fillet area and that needs the shape of non-standard tool tip to be “flatter” than the standard one. It is essential to verify if it is even possible to construct such a tool shape.

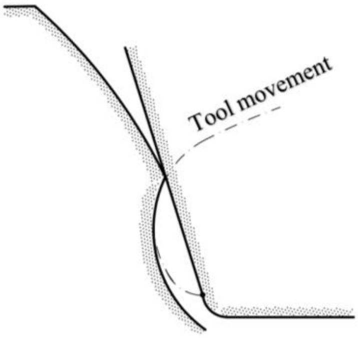

Figure 8. Undercutting by hob

Construction of a non-standard tool shape is an inverse task to a tooth profile construction. In this case, the tooth profile with non-standard root shape is known and the shape of tool tip needs to be obtained. For purpose of this study, the tool shape is constructed using “direct construction” graphical method [Salamoun 1990]. Tool profile for gear of parameters according to Tab.1 with the elliptical root fillet is possible to construct and is shown on Fig. 9. For gear with the cycloidal root fillet results are analogical.

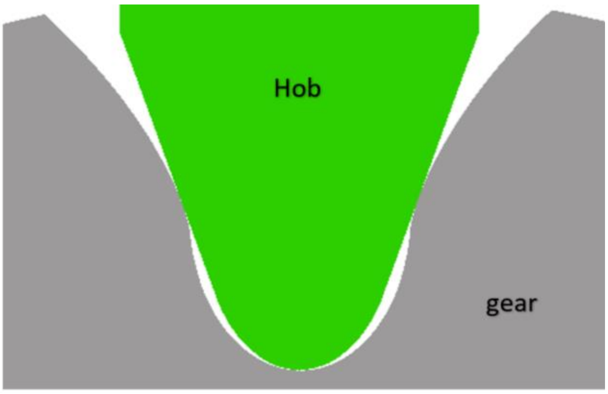

Figure 9. Tool profile of non-standard fillet

4.2 *Finishing process*

Commonly used method for tooth surface finishing is grinding. A rough tooth shape is usually produced by hobbing and subsequently the functional part of a tooth profile is finished by grinding. In other words, only the involute part of tooth is fully grinded and the border between hobbed and grinded surfaces lies on the root fillet. This transition is not smooth and thus acts as a notch. In standard tooth shape, in order to minimise the notch effect, the protuberance is usually used. However, that means more material is removed from the root fillet. Using nonstandard root fillet leaves more material in that area, and that cause the transition angle between both surfaces to be smaller, furthermore the notch moves closer to the root circle as shown in Fig. 10. As a result, the notch effect is more significant, and moreover located in root filet section with higher stress value. This applies to comparison of gears of same parameters (differ just by root filet shape) and finished by same grinding tool.

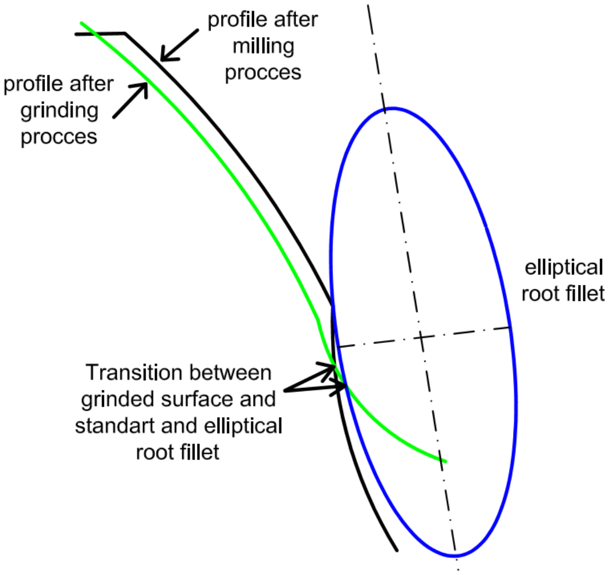

Figure 10. Finishing process transition points

5 CONCLUSIONS

Tooth profile of representative undercut gear of selected parameters was defined with standard and nonstandard elliptical and novel cycloidal root fillet. These fillets were compared between each other, and results show that the elliptical root fillet brings 13% decrease of root stress compared to standard rounded filet, for the cycloidal fillet the improvement is even more significant: 30% according to FEM. These results were verified by analytical ISO model that shows decrease of 14% for the ellipsis and 32% for the cycloid that roughly corresponds with FEM results.

From the manufacturing possibilities point of view, it was verified that tool with nonstandard tip is possible to define and such a gear can be produced by conventional hobbing method that makes it applicable in mass-production. Regarding finishing processes, it was found out that using nonstandard fillets together with a traditional grinding tool shape leads to a greater notch in transition point. This means that another approach to grinding tool shape should be considered and opens a new space for investigation.

Practical application of these nonstandard fillets could be found in cases when centre distance and gearbox construction dimensions are defined, and gear ratio needs to be modified, while preserving its original tooth strength. This kind of problems could be occasionally found in automotive.

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