THE USE OF GEOGEBRA IN TECHNICAL MATHEMATICS

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DOI: 10.17973/MMSJ.2023_03_2022112

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In many technical fields, it is necessary to have a basic set of mathematical skills. Engineers need to calculate and understand concepts. They have to know how to solve equations, analyse measurements, use tables and graphs for some processes. These are just some examples showing how mathematics learning is important. For better illustration and visualization of mathematical concepts and methods, it is appropriate to use a modern technology. This provides dynamic teaching opportunities in math classes. GeoGebra is a free math software that is very useful for this purpose. This article shows some practical examples of the GeoGebra usage both as in mathematics education and as a support in solving technical problems.

KEYWORDS

mathematics, physics, GeoGebra, modelling, education

1 INTRODUCTION

GeoGebra is an interactive mathematics software suitable for learning and teaching science, technology, engineering, and mathematics from primary school up to the university level. Constructions can be made with points, vectors, segments, lines, polygons, conic sections, inequalities, implicit polynomials and functions, all of which can be edited dynamically later. Elements can be entered and modified using mouse and touch controls, or through an input bar. GeoGebra can store variables for numbers, vectors and points, calculate derivatives and integrals of functions, and has a full complement of commands like Root or Extremum. Teachers and students can use GeoGebra as an aid in formulating and proving geometric conjectures [wikipedia.org].

Application is available on multiple platforms (Windows, Linux, Android, iOS, macOS, etc.) and it is also possible to run it directly in a web browser. Users also appreciate multiple language mutations. GeoGebra is a community of millions of users located in just about every country. It has become the leading provider of dynamic mathematics software, supporting science, technology, engineering and mathematics (STEM) education and innovations in teaching and learning worldwide. GeoGebra's math engine powers hundreds of educational websites worldwide in different ways from simple demonstrations to full online assessment systems [geogebra.org].

The official GeoGebra website, www.geogebra.org, provides the latest version of the software for download, access both to GeoGebraWiki and to forum and materials area. Materials area of Geogebra's website is a site for sharing GeoGebra materials. Materials can be easily searched and are also to available to unregistered users.

The International GeoGebra Institute (IGI) is the nonprofit arm of the GeoGebra Group. The institute coordinates research, development, translation and deployment efforts of the GeoGebra system across a global network of user groups at universities and non-profit organizations, as well as provide certification to GeoGebra experts and trainers [wikipedia.org]. IGI is an umbrella organization for local GeoGebra institutes. Local GeoGebra Institutes join teachers, students, software developers and researchers from around the world. Nowadays, there are over 40 institutes in more than 30 countries.

GeoGebra Institute of Ostrava (ggi.vsb.cz), founded in 2016, is hosted by the Department Mathematics and Descriptive Geometry, VSB-Technical University of Ostrava.

Main objectives of our institute in Ostrava are to:

1. promote usage of GeoGebra resources
2. organize seminars and workshops for teachers and students from primary schools to university level
3. create GeoGebra applets and materials and share them with others.

2 GEOGEBRA IN THE MATH LESSONS

Currently, the application of new teaching and learning methodologies in mathematics is a highly discussed topic. The fact that teachers work together to plan, teach and reflect on a lesson can play an important role in improving the quality of teaching and learning at this academic level. The paper [Caridade 2022] contains the opinion and reflections of a group of teachers on the effect of their participation in a Lesson Study. Lesson Study (LS) is a professional development process for teachers created in Japan. The chosen topic was the Newton-Raphson method for solving nonlinear equations using GeoGebra as dynamic content.

GeoGebra is of interest for educators because it is an open source software that can be an effective tool to teach geometry, calculus, algebra, and statistics. A team of authors leads basic subjects of mathematics, numerical methods and statistics at the Faculty of Mechanical Engineering. Mathematics courses mainly concern differential and integral calculus. The primary objective of teaching is to prepare students for studying practical subjects so that students are able to use basic mathematical concepts and methods appropriately. In particular, students need to know how to use the derivative to describe dynamic processes and should be able to perform data analysis. We therefore focus on examples of using GeoGebra to explain these concepts and methods.

The same approach can also be used in the teaching of other science subjects like chemistry, biology, and physics. Specific topics how GeoGebra can be used in the teaching of science subjects are listed in the section 3.2.

2.1 Differential calculus

A visualization of all terms is necessary for a correct understanding of study topics. The derivative is one of the crucial terms in mathematics courses. The geometrical meaning of derivative is the slope of tangent in a given point. This can be visualized in three easy steps:

— to plot a graph of the function *f*

— to place a point *A* anywhere on the graph

— to construct a tangent by using the command *Tangent(A,f)*

The slope of tangent line can be shown by defined tool in menu, see Fig. 1.

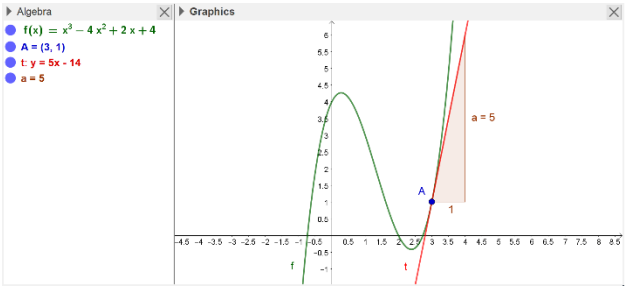


Figure 1. The tangent line visualization

If the point *A* is moving on the graph of the function *f*, the change of the tangent and its slope is immediately displayed. We discuss the meaning of derivative with students. The value of the slope is dynamically plotted in the second graphics view. It is useful for better understanding of the subject and students immediately see the relation between function and its derivative (Fig. 2).

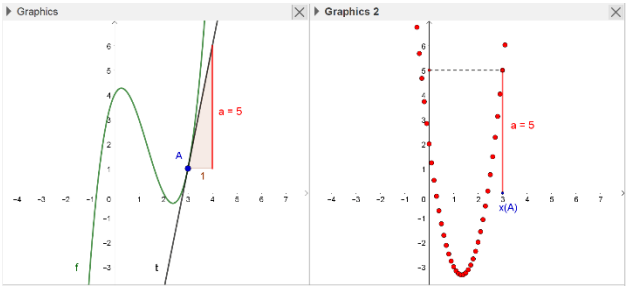


Figure 2. The values of tangent line slope in some points

GeoGebra does not only help with visualization but can also assist with computing. Taylor series are powerful tools for approximating functions that can be difficult to compute. Fig. 3 shows an advantage of GeoGebra for explaining this term. It is possible to use Graphics view to plot a graph of a given function with the determined Taylor polynomial and at the same time use the Spreadsheet view to display the calculated functions values. It is very easy to increase the degree of the Taylor polynomial, which leads to a better approximation of the function around a point.

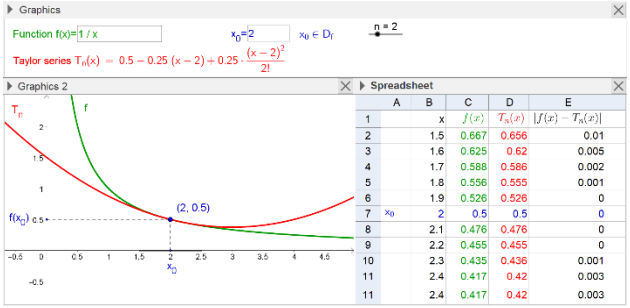


Figure 3. Approximation of a function by a Taylor polynomial

An inherent part of the engineering math education is a knowledge of differential equations. Students should know how to find an analytic solution of an equation, but they also should understand the graphical meaning of a differential equation showing by slope field. The first-order differential equation is an equation of the form

$$y' = f(x, y).$$

The function defined on the right side of this equation is inserted into the Input Bar in GeoGebra. We create the object, and it is graphically displayed in the Graphics view. The slope field is plotted by command *SlopeField(f)*. It is often needed to see integral curves of an initial value problem. To display the integral curve of the initial problem, we input the initial value $y(x_0) = y_0$ represented by point $A = (x_0, y_0)$ and draw the particular solution with *SolveODE(f,A)* command. The trace of the integral curve can be shown by setting "Trace on" in the context menu of this curve (Fig. 4).

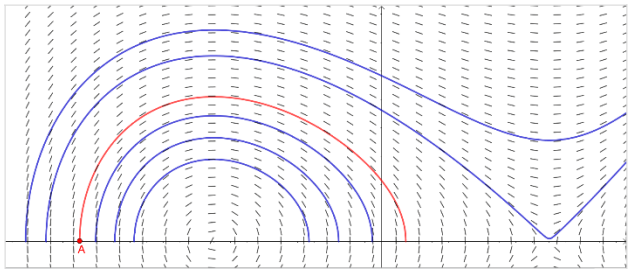


Figure 4. Slope field and some solutions of differential equation

Topics that require a graphical 3D representation are an important part of our math courses. GeoGebra is very helpful here because it contains a 3D-graphics view. A connection between 2D and 3D graphics view is beneficial for a problem of finding local maxima and minima of two variables function subject to equality constraints, see Fig. 5.

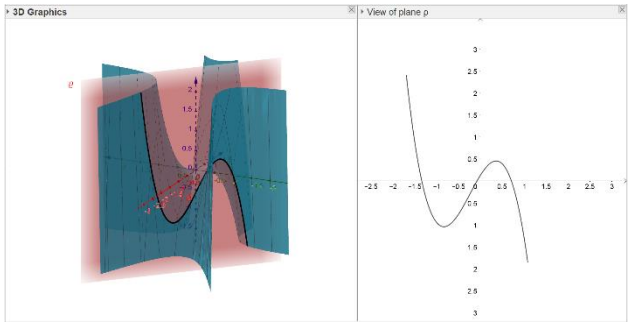


Figure 5. The local maxima and minima of two variables function subject to equality constraints

2.2 *Statistics*

In technical practice, it is very often necessary to perform basic data analysis — descriptive statistics, estimation of errors and means, normality test, comparison of data sets, analysis of relationships between variables. GeoGebra includes tools to analyse statistical data and exploring the probability distribution functions.

In GeoGebra, there are two views useful for statistics, Spreadsheet and Probability Calculator. The Spreadsheet view is very similar to Excel. Like in it, we can determine basic numerical characteristics by defined functions. The Spreadsheet toolbar contains GeoGebra's tools for statistics (One Variable Analysis Tool, Two Variable Regression Analysis Tool, Multiple Variable Analysis Tool), tools for statistical graphs (histogram, dot plot, box plot) and means to find numerical characteristics (mean, quartiles, standard deviation, etc.). It is possible to connect the data to the Graphics view and use it for an illustrative interpretation (Fig. 6). A big advantage is that in case of data changes, all calculations and graphs are modified immediately. It is very easy to modify the output, insert text, change the colours and parameters of the graphs, etc.

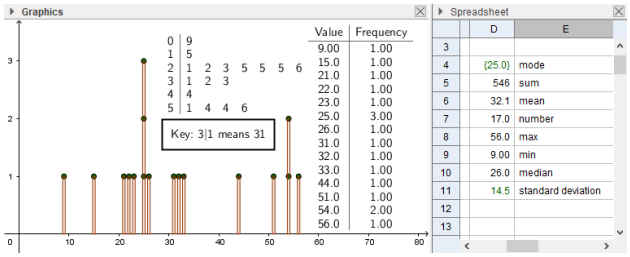


Figure 6. Numerical characteristics and plots

Another helpful part of GeoGebra is a Probability Calculator. This one can be used to calculate and graphically represent different probability distributions (Fig. 7), as well as to perform statistical tests and confidence intervals estimation. The "Statistics" option allows to perform statistical tests in an easy way. GeoGebra automatically displays results of a statistical test, after a specified test and hypothesis.

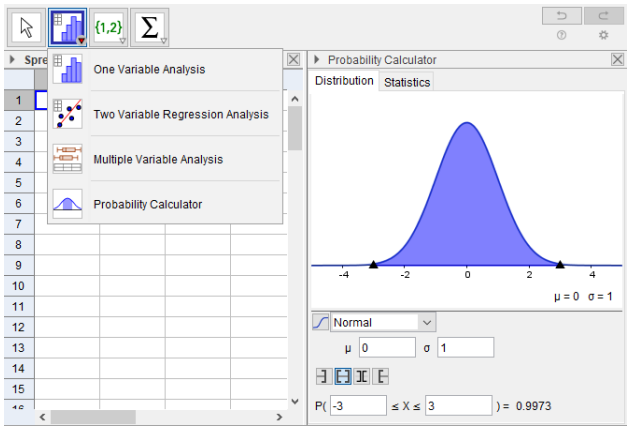


Figure 7. Statistics tools and normal distribution in the Probability Calculator view

There are two different ways of using GeoGebra for statistics courses.

1. GeoGebra applets to demonstrate concepts (confidence interval, normal distribution, least squares method, etc.), Fig. 8 shows the applet that demonstrate the meaning of parameters for the construction a confidence interval about the population mean.

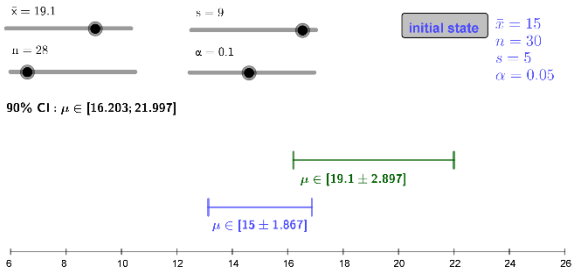


Figure 8. GeoGebra applet for construct a confidence interval

2. Students use GeoGebra to perform data analysis (numerical characteristics, graphs, analysis of variance, statistical tests, linear regression — Fig. 9).

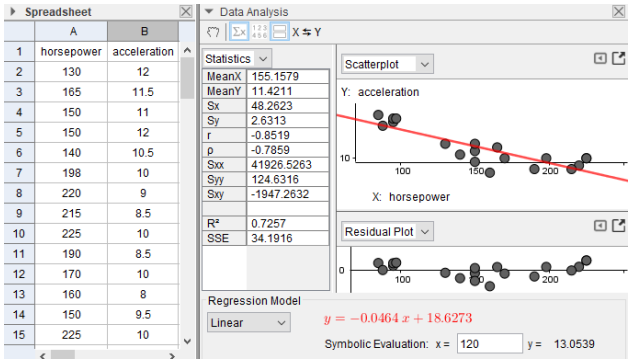


Figure 9. Output for regression and correlation analysis

In [Phan2018], authors discuss the usage of GeoGebra as a tool to collect and analyze data from a geometric estimation activity and they show that this software also includes powerful tools for statistical analysis.

3 TECHNICAL APPLICATION

GeoGebra is supportive also for engineers in many technical fields.

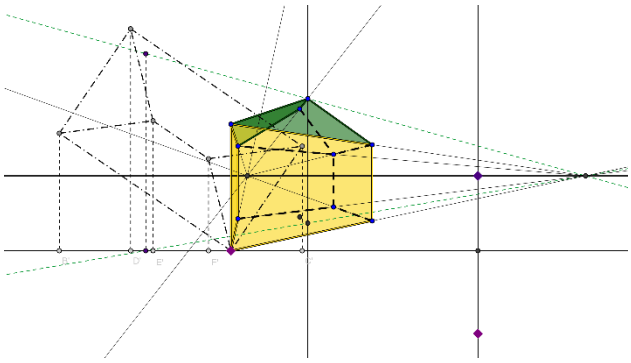


Figure 10. Architectural drawing - hip roof in corner perspective

For example, situations where it is necessary to graphically represent curves or surfaces based on corresponding equations. For this, the algebraic and geometric tools included in the GeoGebra software can be easily used. GeoGebra is an attractive alternative to other tools that are used in practice for designing and modeling curves and solid objects [Falcon 2010]. Its advantages are an intuitive control, an easy way to enter mathematical formulas, equations, the usage of sliders and projection from 2D to 3D. A modeling in 3D allows different views, rotations, displacements together with application in technical drawing in civil engineering is seen in [Mihailova 2014].

[Ziatdinov 2022] describes a usage of GeoGebra and its effectiveness in modeling applications and situations where analytical geometry, algebra and visualization are needed.

GeoGebra's interactive geometry framework, as a drawing application for azimuthal equidistant spherical perspective, is described in [Araujo 2020]. The presented tool is handy for researchers.

In various fields, the action, and dynamic changes of states in applications are modeled by differential equations. GeoGebra will help to find their solution and immediately display or apply a result. [Olivares 2020] shows a process of finding a solution of linear homogeneous differential equations of third and fourth order.

3.1 *Construction of a harmonic motion*

Typical problems in mechanics, that can be formulated using linear differential equations, also include oscillatory processes. We show how to use GeoGebra (Fig. 11) to perform a basic analysis for mechanical oscillations and to find a solution for the harmonic motion defined by the equation

$$y'' + ay' + \frac{k}{m}y = 0,$$

a, k, m are constants, with initial conditions

$$y(0) = y_0, \qquad y'(0) = y_1.$$

The applet shows a solution for an oscillating system. It is convenient to use two Graphics views and 3D Graphics view. In 2D Graphics views, we define a problem and find a solution. In a second one, there is a start/stop button for simulating and displaying of a temporal behaviour of a displacement in time and simultaneously a motion is animated in a 3D view.

All of parameters can be altered using sliders. By changing parameters, one can immediately see that if a damping is strong compared to the spring constant, a motion is overdamped, but if this damping is small, the motion oscillates.

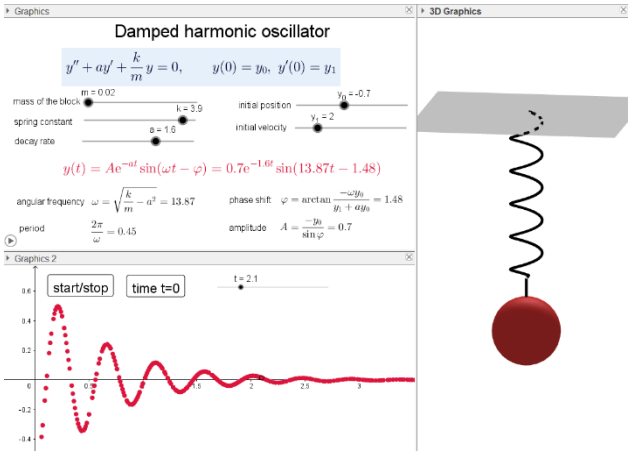


Figure 11. Harmonic motion applet

3.2 *Examples of GeoGebra Science applets*

In this section, we want to introduce GeoGebra applets that can be utilized in various technical parts. It is mostly used to describe and to model physical problems and tasks. GeoGebra provides a simple way how to simulate motion and dynamics. Using sliders, input parameters can be changed very easily, and this change will be displayed immediately. A few examples follow: using the principles of kinematics, dynamics, and other physics fields (Fig. 12-15).

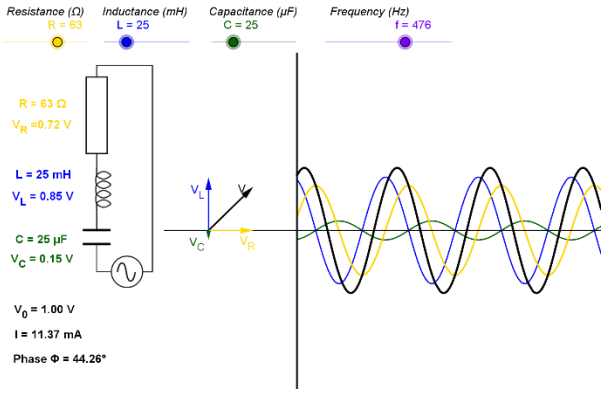


Figure 12. The operation of a series inductor-capacitor-resistor circuit

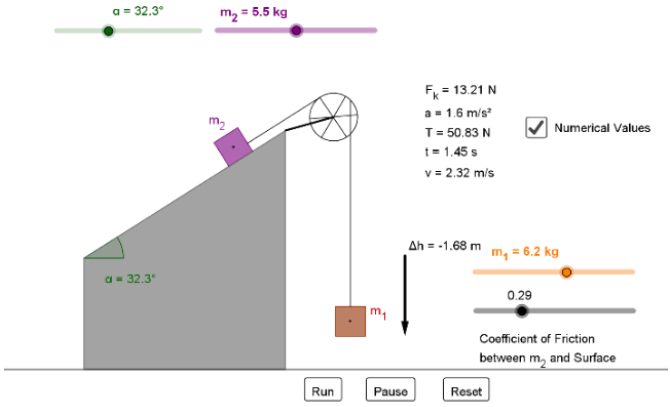


Figure 13. Motion on an incline

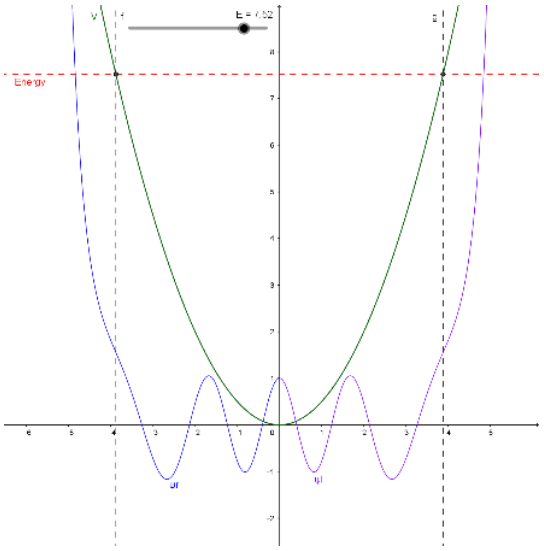


Figure 14. The numerical solution of Schrödinger's equation (quantum mechanics)

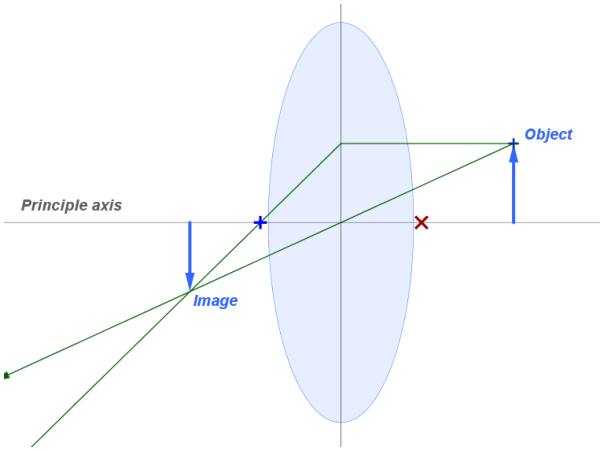


Figure 15. A ray tracing diagram for a converging lens which follows the opposite convention

Software GeoGebra and its 3D environment is a very helpful for visualization of many applications and makes a strong connection between real life problems and their visual representations. For instance in chemistry courses, it is possible to create a 3D representation of simple molecular structures (Fig. 16, geogebra.org, activity FKjmg4Sp). Another example is a problem solving in the area of quantitative composition of solutions (QCS).

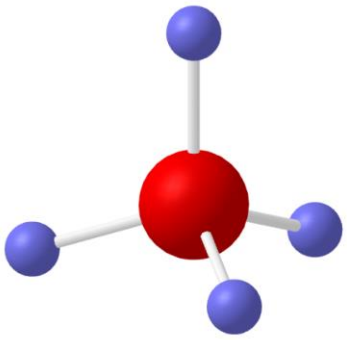


Figure 16. The methane molecule 3D model

GeoGebra enables an export of 3D models in stl format. Therefore, it is very suitable for modelling objects in 3D printing.

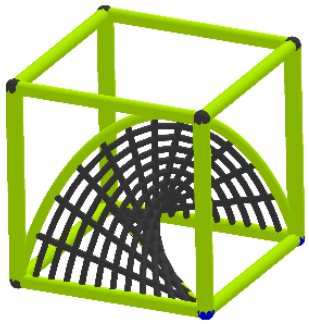


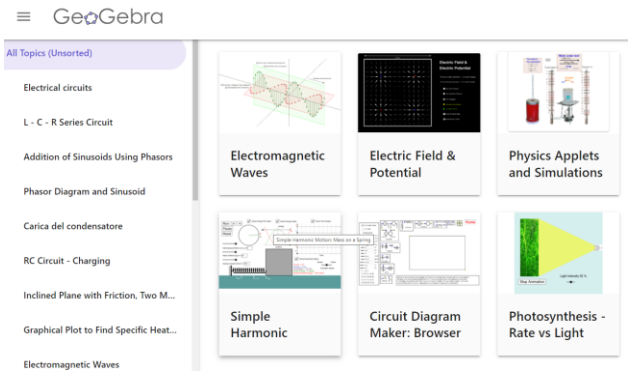
Figure 17. 3D printable model

Additional materials are available at GeoGebra books (geogebra.org, 08/2022). Following some examples and links to interesting GeoGebra books. There are also supply by screenshots of selected topics.

1. Collection of Science applets, simulations. Physics, Chemistry Biology

author: Lew W. S.

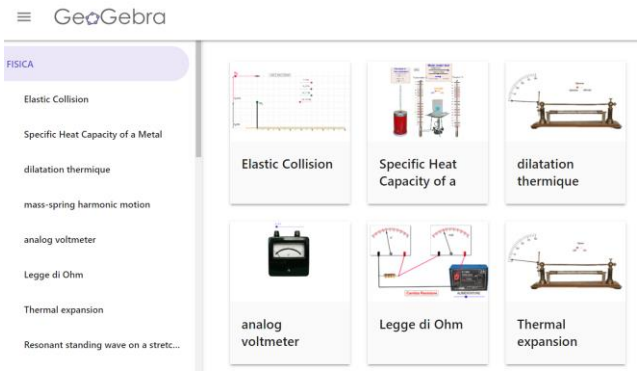
https://www.geogebra.org/m/g444wWSd



2. FISICA

author: Luciano Troilo

https://www.geogebra.org/m/Z57h2sQc

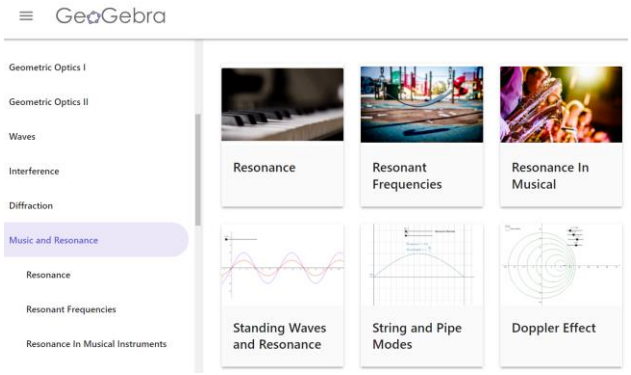


3. Interactive Engineering Physics with Numerical Methods

It is an open-source, comprehensive engineering physics textbook for the first three semesters of a degree in engineering, physics or chemistry. An emphasis is placed on fundamental principles as well as numerical solutions to equations where no analytical solution exists.

author: SAVAŞ ORHAN, Timo Budarz, Sierra Brown

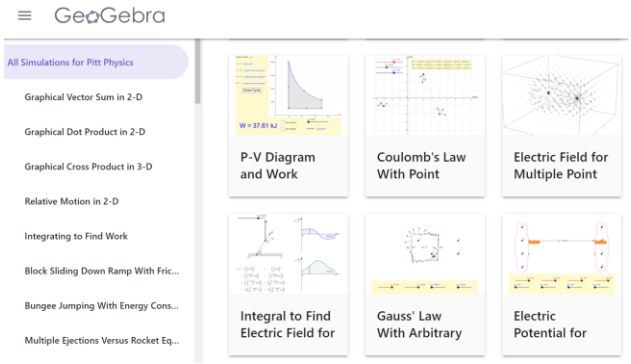
https://www.geogebra.org/m/Y38XGs9F



4. All Simulations for Pitt Physics

author: Dave Nero

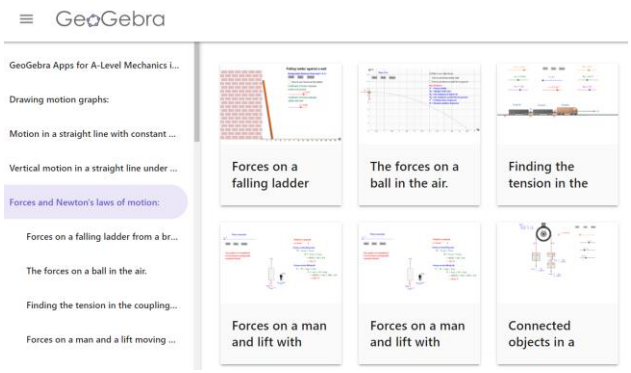
https://www.geogebra.org/m/nk83G3WY



5. GeoGebra Apps for A-Level Mechanics including FM

author: Mark Willis

https://www.geogebra.org/m/regzpujq

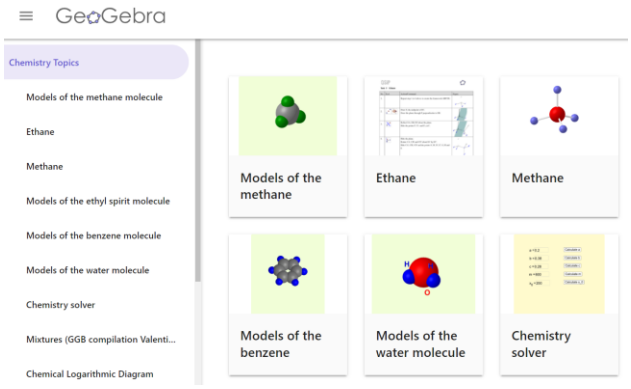


6. Chemistry Topics

Collation of chemistry resources in GeoGebra.

author: W.S. Lew

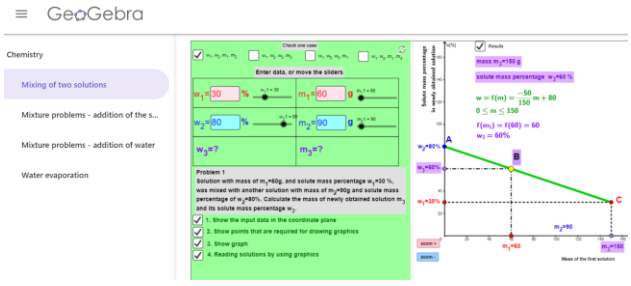
https://www.geogebra.org/m/vntCrpmd



7. Chemistry

author: Valentina Kostic

https://www.geogebra.org/m/C4M42dr7



8. GED Science

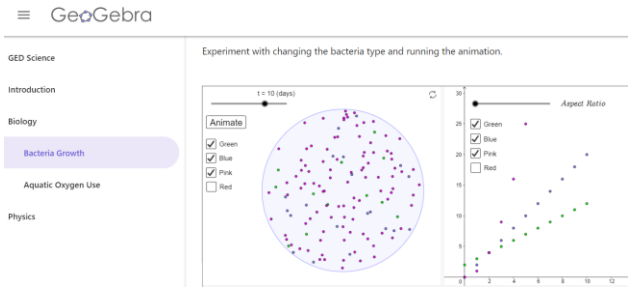
General Educational Development (GED) tests are a group of four subject tests which, when passed, certify that the test taker has American or Canadian high school-level academic skills.

author: A. B. Cron

https://www.geogebra.org/m/kH5xMUpw

Application in Biology

Bacteria grown: Experiment with changing the bacteria type and running the animation.



4 CONCLUSIONS

In this article, we presented how to use GeoGebra in a teaching of mathematics and statistics, as well as in various technical applications. GeoGebra is a complex and dynamic mathematical program that can be helpful in various mathematical calculations and tasks. It is easy-to-use dynamic mathematics software, that combines many aspects of different mathematical packages, and it is not complicated to navigate through and is intuitive to use. GeoGebra is very user-friendly and offers many advantages in fields of calculus, algebra, modelling, physics, technical drawing, and linear programming, among many others. Because of its open-source nature, an extensive user community has developed around it and GeoGebra software has been gaining popularity among teachers and researchers around the world.

Users can easily manipulate variables by simply dragging free objects or using sliders. It is therefore possible to generate changes and then analyse how objects are affected by these changes. Users thus have the opportunity to solve problems dynamically.

It has a potential to bring an interaction and visualization in learning mathematics and science, through its dynamic features. A usage of applets created with GeoGebra has a positive effect on students' understanding and knowledge.

We also showed the possibility of using ready-made materials and applets that are freely available on the GeoGebra's website. Existing materials can be downloaded and modified as needed. GeoGebra is a user-friendly software that can be controlled intuitively and offers countless possibilities of use not only in teaching, but also in technical practice.

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