DESCRIBING STATISTICAL DEVIATIONS OF PROTECTION TIMES OF LASER SAFETY BARRIERS

FLORIAN P. LUGAUER, FABIAN WIMMER, MICHAEL F. ZAEH

iwb - Institute for Machine Tools and Industrial Management, Technical University Munich

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e-mail: florian.lugauer@iwb.tum.de

Passive laser barriers are designed to protect humans from risks of laser radiation in the production environment. Manufacturers of such barriers have to guarantee the protective effect. Therefore, experiments are conducted to determine their protection times. For this purpose, some sample based approaches exist but they are not statistically reliable. An approach for the description of the distributions of protection times of passive laser safety barriers is presented to develop a statistical reliable method for the estimation of safe protection times out of few samples. The analysis result of the statistical deviations shows that the measured data can be described by a bimodal approach of two weighted normal distributions.

KEYWORDS

laser safety, IEC 60825-4, industrial safety, protection time distribution, employment protection

1 INTRODUCTION AND STATE OF THE ART

The number of laser applications has been increasing in manufacturing [Optech Consulting 2015]. Due to this growth, the aspect of laser safety has become more important. Passive laser housings represent a common measure to protect humans against the risks of laser radiation. In this context, the laser protection time t_s is defined in this work as the period of time from the beginning of irradiation until the laser beam melts the material of the barrier and the radiation finally penetrates it. To test the barriers on-site, a mobile test rig has been developed at the *iwb* [Lugauer 2014]. Using this rig, statistical deviations of protection times t_s were determined for several materials. Considering the high financial and time expenses of such experiments, the challenge is to estimate protection times t_s out of few samples. Various methods exist on how to estimate a nominal protection time t_s. In the relevant standard IEC 60825-4, the protection time t_{s1} is calculated according to equation 1 [IEC 2006]:

$$t_{s1} = 0.7 \cdot \left(\bar{x} - 3 \cdot s\right) \tag{1}$$

$\bar{x}$ represents the sample mean and s the standard deviation of the sample. The procedure is based on the assumption that laser protection times are normally distributed. A sample size of $n \geq 6$ is referred by the standard. Besides the proposal in equation 1, there is another approach to calculate protection times t_s The nominal laser protection time t_{s2} is determined based on a sample size of n = 10 [DGUV 2013] by equation 2:

$$t_{s2} = 0.7 \cdot \min \ x_i \tag{2}$$

In order to estimate the nominal protection time t_{s2} using the second approach, the smallest empirical value of the sample min x_i has to be multiplied, like in the first approach, with a safety factor of 0.7. It is noteworthy that the distribution function of the protection times is not being considered with the second approach.

Therefore the questions arise, which approach is a valid one, how are protection times of several materials distributed and can the protection time of a laser safety barrier t_s be determined by means of few samples. In order to find a proposal that is based on reliable statistical methods, experimental data of the distribution of laser protection times with different materials, radiation intensities I and a varying number of samples n were analysed. Within this work, a possibility to describe statistical deviations of laser protection times using a bimodal approach of description with a sum of two weighted normal distributions, kernel density estimation and least-squared-optimisation is shown.

2 ANALYSIS AND INTERPRETATION OF PROTECTION TIME TEST RESULTS

To find a suitable approach to describe statistical variations of protection times, empirically determined test results were analysed and interpreted. The data have been collected by experiments that have been carried out at the *iwb* using the setup described in [Lugauer 2014]. A quite large number of samples n is available, which leads to a more detailed knowledge of the distribution. The laser power P_L, the diameter of the laser spot at the plate surface d_L, the number of samples n, as well as the material of the plates were varied. In addition to the steel alloys 1.0241 (zinc-magnesium coated) and DC-01, the aluminum alloy EN-AW-5083A was tested. The dimensions of the samples were 280 x 280 x 1.5 mm. An overview of the experiments, the parameters and the particular sample size n is given in Tab. 1.

To illustrate the further proceeding, the next steps are exemplary demonstrated at test series 3, which protection times are shown in Fig. 1.

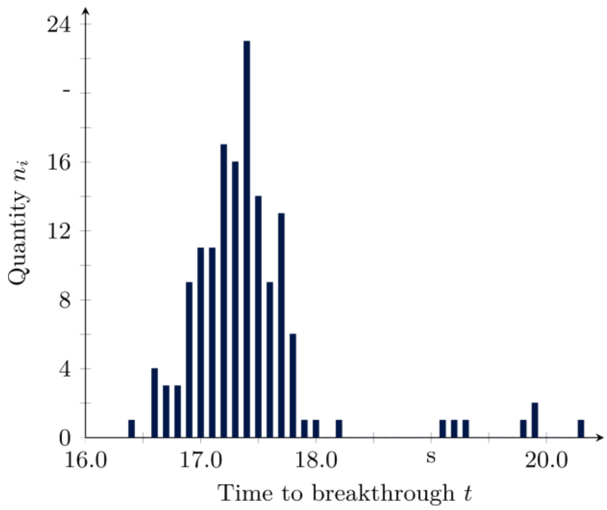

Figure 1. Statistical distribution of protection times of zinc-magnesium coated steel 1.0241 samples irradiated with a Ytterbium fibre laser at a laser power P_L = 3 kW and with a beam diameter of d_L = 60 mm. Sample size n = 150. [Lugauer 2015]

For further analysis, a histogram of each dataset is created with $\sqrt{n}$ classes of equal width and a linear transformation is applied to each value in order to achieve a consistent and comparable representation. For the latter mentioned purpose, equation 3 was used [Verma 2015]:

$$\tilde{x}_i = \frac{x_i - \bar{x}}{s} \quad (3)$$

The theoretical probability density function assuming a normal distribution was calculated based on the mean of the sample $\bar{x}$ and the standard deviation of the sample s. The result can be seen in Fig. 2.

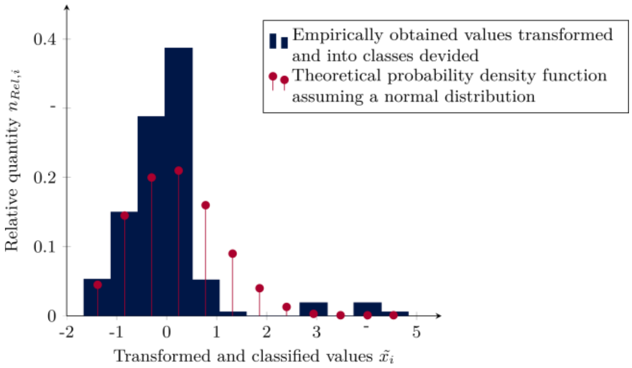

Figure 2. Empirical values of Fig. 1 transformed and divided into classes in comparison with the theoretical probability density function assuming a normal distribution.

A visual comparison between the empirical values and the theoretical probability density function reveals a clear disparity: The theoretical maximum value is much smaller than the empirical value and the dispersion of the theoretical values is larger than that of the experimental values. In addition, a test of goodness of fit in view of a normal distribution was applied. A suitable test for this purpose is the Anderson-Darling test, firstly described in 1952 [Anderson 1952], [Stephens 1979]. The calculation was executed by the aid of the MATLAB Statistics and Machine Learning Toolbox, more precisely the script ´adtest´ [Mathworks.2016]. The resulting p-level implies a significant deviation of the empirical values from the normal distribution if $p_{AD} < 0.05$ [Anderson 1952], [Stephens 1979], [Mathworks 2016].

Table 1. Experiments carried out to determine the distribution of protection times. (P_L = laser output power, d_L = beam diameter at the sample surface, n = sample size, p_{AD} = p-value of the Anderson-Darling test, p_{HD} = p-value of Hartigan's dip test, R = residual of the bimodal approach with two normal distributions, R_{chy} = residual of the bimodal approach with a normal and a Cauchy distribution, R_{wb} = residual of the bimodal approach with two Weibull distributions)

Nr	material	P_L in kW	d_L in mm	n	p_{AD}	p_{HD}	R	R_{chy}	R_{wb}
1	1.0241	3	40	50	0.0035	0.00	0.03	0.03	4.40
2	1.0241	3	60	50	0.2450	0.52	0.22	0.22	1.24
3	1.0241	3	60	150	1.6×10^{-5}	0.00	0.11	1.88	0.24
4	1.0241	3	80	50	0.8270	0.84	0.40	0.41	0.04
5	1.0241	5	40	50	0.0018	0.00	0.00	0.10	1.65
6	1.0241	5	60	49	0.0217	0.00	0.11	0.17	1.15
7	1.0241	5	80	50	0.0150	0.42	0.07	0.09	2.13
8	1.0241	5	80	150	6.43×10^{-6}	0.00	0.18	1.07	0.12
9	1.0241	7	40	50	0.0036	0.00	0.04	0.53	0.59
10	1.0241	7	60	50	0.0160	0.00	0.08	0.36	2.84
11	1.0241	7	80	50	0.0210	0.00	0.03	0.25	0.79
12	1.0241	7	80	150	0.0056	0.00	0.50	0.36	0.03
13	DC-01	3	60	50	0.0050	0.00	0.04	1.45	5.19
14	EN AW-5083 A	5	40	50	0.4050	0.00	0.07	0.16	0.09

As shown in Tab. 1 for most of the data sets the null hypothesis, which states that the examined values correspond to a normal distribution must be rejected. In five cases, p_{AD} is larger than or equal to 0.05, which means on the one hand that the null hypothesis cannot be rejected, but on the other hand the protection times may not inevitable correspond to a normal distribution.

A deviation between the statistical distribution of protection times of laser barriers and the normal distribution was already discussed and mentioned in several works, e. g. [DGUV 2013] and [Lugauer 2015]. The fact that empirical protection time values differ in particular left of the area from the maximum of the normal distribution is often discussed among experts, however, was never fully investigated. Zaeh & Braunreuther used a Weibull-distribution to describe the statistical behaviour of protection times of hollow chamber barriers [Zaeh 2010]. Due to the complex determination of the Weibull-parameters, they finally used a normal distribution as approximation. As shown above, this approach is not suitable for the description of the considered data sets of the present work. Therefore, another procedure was chosen: The subjective observation of the empirical data indicates that the protection times may be distributed multimodal. `The mode is a measure of central tendency. The mode of a set of observations is the value of the observation that have the highest frequency. According to this definition, a distribution can have a unique mode (called the unimodal distribution). In some situations a distribution may have many modes (called the bimodal, trimodal, multimodal, etc. distribution)´ [Dodge 2008]. The simplest form of multimodality is bimodality, which means that an examined probability distribution shows two distinct peaks respectively local maxima. Freeman & Dale mention inter alia Hartigan's dip test (HDT) as suitable means for testing a distribution on bimodality [Freeman 2012].

The HDT, firstly described by Hartigan & Hartigan in 1985, employs the null hypothesis that the observed distribution is unimodal [Hartigan 1985]. If the calculated p-value p_{HD} of the HDT is smaller than 0.05, a significant bimodality is indicated, and the null hypothesis has to be rejected [Freeman 2012]. Values between 0.05 and 0.10 suggest a bimodality with marginal significance [Freeman 2012]. The HDT was applied to the data sets at hand using Mechlers MATLAB translation of Hartigans original FORTRAN subroutine [Mechler 2002].

The results are shown in Tab. 1 rounded to two decimals: For most of the examined data sets, p_{HD} lies far beyond 0.05 and therefore the HDT indicates bimodality.

In summary, the examined empirical values do not show a single normal distribution but rather a bimodal distribution.

3 DESCRIBING PROTECTION TIMES OF LASER SAFETY BARRIERS

The investigations of section 2 revealed a bimodal distribution of protection times and because of the wide spread of the normal distribution in nature and technology and its good handling quality the following approach of description was chosen:

$$F\left(q,\mu_1,\sigma_1,\mu_2,\sigma_2\right)= q\cdot N_1\left(\mu_1,\sigma_1\right)+\left(1-q\right)\cdot N_2\left(\mu_2,\sigma_2\right) \quad (4)$$

F, the description of the whole distribution, arises as the sum of the two weighted normal distributions N_1 and N_2, which are depending on their particular means μ and standard deviations σ. The weighting coefficient is given by q. To describe a data set at hand, the unknown parameters have to be specified.

For this purpose a spline interpolation was used, which was applied by the aid of the MATLAB function `interp1´ [Mathworks 1996] and the script shown in section 7. Thereby first of all a function including the central values of each class of a particular data set is determined. An exemplary result for the values of Fig. 2 is shown in Fig. 3. Moreover, Fig. 3 displays the bimodal approach with two weighted normal distributions using the calculated parameters for the presented empirically obtained values.

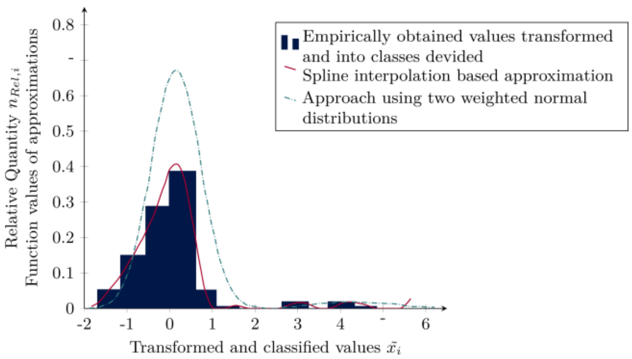

Figure 3. Approximation of empirical protection time values by the aid of a spline interpolation and description of the data using a bimodal approach with two weighted normal distributions.

It is obvious, that the description by the spline interpolation function (SIF) is very accurate and much better than the theoretical probability density function of Fig. 2. In order to get the necessary values for equation 4, the maxima of the SIF were determined. Their values served as an approximation for the searched means μ_1 and μ_2 of the normal distributions. If there were more than two maxima, the two largest were used. The standard deviations σ_1 and σ_2 were estimated based on the curvature of the spline interpolation function at the positions of the local maxima as shown in section 7. The weighting factor q was calculated using the proportion Z of the two function values F (μ_1) and F (μ_2):

$$Z=\frac{F\left(\mu_1\right)}{F\left(\mu_2\right)}=\frac{q}{1-q} \rightarrow q=\frac{Z}{1+Z} \quad (5)$$

The dashed graph in Fig 3 shows the bimodal approach with two weighted normal distributions using the calculated parameters as mentioned above for the presented empirically obtained data. It is evident, that the bimodal description matches the available data much better than the unimodal description shown in Fig 2.

4 IMPROVEMENT OF THE DESCRIPTION USING A KERNEL DENSITY ESTIMATION

In the following, the description by the aid of a bimodal approach with two weighted normal distributions was improved by using a kernel density estimation (KDE). The KDE, described by Parzen, is a method for estimating the probability distribution of a random variable [Parzen 1962]. The aim is to determine the density function $P_{KDE}(x)$ on basis of a finite number of experimental data x_i. $P_{KDE}(x)$ is defined as a weighted sum [Parzen 1962]:

$$P_{KDE}\left(x\right)=\frac{1}{nh}\sum_{i=1}^{n}K\cdot\left(\frac{x-x_i}{h}\right) \quad (6)$$

In equation 6, x is a continuous random variable, h is the bandwidth of the KDE with $h > 0$, n is the sample size and x_i is the value of a single measurement. The bandwidth h is a selectable parameter, with influence on the quality of the estimation. The theorem of Nadaraya says that with an appropriately chosen bandwidth an arbitrarily good estimate is possible [Nadaraya 1964], [Nadaraya 1965]. The kernel K is a function, which assigns the continuous random variable x a value in the vicinity of the measured value x_i. Several functions can be used for the kernel, a common used one is the normal distribution. In this case, the kernel can be described as [Parzen 1962]:

$$K\left(\frac{x-x_i}{h}\right)=\frac{1}{\sqrt{2\pi}}\cdot\exp\left(-\frac{1}{2}\left(\frac{x-x_i}{h}\right)^2\right) \quad (7)$$

Vividly illustrated, the KDE utilising the normal distribution is built up as an integrated sum of particular normal distributions, which spread around each measurement x_i.

Since the KDE can be seen as an ideal description of the empirical values, the bimodal description of section 3 can be improved by adjusting the parameters in a way that the difference D between KDE and F of equation 4 is minimal. This is an optimisation task, which can be solved using the method of least squares [Rawlings 1998]. D can be written as:

$$D=\sum_{x}\left[P_{KDE}\left(x\right)-F\left(q,\mu_1,\sigma_1,\mu_2,\sigma_2\right)\right]^2 \quad (8)$$

The optimisation procedure needs an initial estimation for the parameters of F. For this purpose, the values determined by the usage of the spline interpolation in section 3 was used, because as shown above, a good approximation was achieved using this method. Fig. 4 shows the description of the empirically obtained data of Fig. 3 employing KDE and the optimised bimodal approach with a sum of two weighted normal distributions:

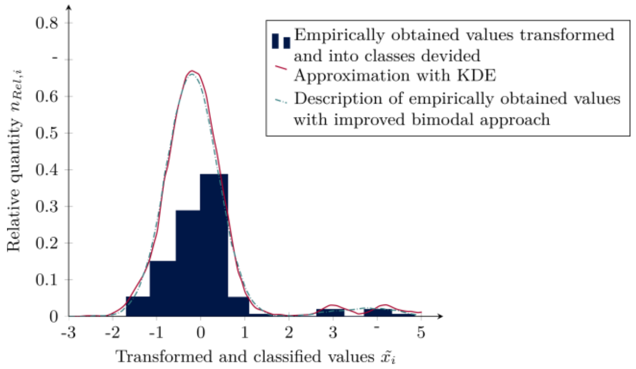

Figure 4. Values of Fig. 3 approximated by KDE and the KDE optimised bimodal approach with two weighted normal distributions

To quantify the quality of the improvement, the residual $\vec{R}$ is calculated as the difference between the values of $P_{KDE}(x)$ and the theoretical function F. So $\vec{R}$ is the sum of the particular differences at all points of discretisation. The residual $\vec{R}$ is a vector. To calculate its length, the Euclidian norm of the residual $||\vec{R}||$ has to be calculated [Rawlings 1998]:

$$R=\left\|\vec{R}\right\|=\sqrt{\sum_x\left[P_{KDE}(x)-F\left(q,\mu_1,\sigma_1,\mu_2,\sigma_2\right)\right]^2} \quad (9)$$

The optimisation and the calculation of R was executed utilising the MATLAB-function `lsqcurvefit´ [Mathworks 2015] and the script shown in section 7. The resulting residuals R are listed in Tab. 1.

Comparing Fig. 3 and 4, it is obvious that the latter mentioned shows a better match of the theoretical and the real distribution. So it can be summarised, that the KDE- and least square-based optimisation leads to a better description of the empirically obtained values than the exclusive application of the spline-based method. The small residuals underline the optical impression of better adapting.

5 ALTERNATIVE APPROACHES

Besides the above-mentioned bimodal approach with a sum of two weighted normal distributions, other sensible approaches are conceivable. So in addition the presented method, approaches with a sum of a normal distribution and a Cauchy distribution and an approximation with a bimodal distribution were calculated in analogy to the explanations of section 4.

The Cauchy probability density distribution can be written as:

$$p_{chy}(x)=\frac{1}{\pi}\cdot\frac{\lambda_{chy}}{\lambda^2_{chy}+\left(x-\mu_{chy}\right)^2} \quad (10)$$

with $-\infty < \mu_{chy} < \infty$ and λ_{chy} as parameters [Prokhorov 2012]. The bimodal approach with a sum of a normal and a Cauchy distribution is:

$$F\left(q,\mu_1,\sigma_1,\lambda_{chy},\mu_{chy}\right)= q\cdot N_1\left(\mu_1,\sigma_1\right)+(1-q)\cdot p_{chy}\left(\lambda_{chy},\mu_{chy}\right) \quad (11)$$

The Weibull probability density distribution can be written as:

$$p_{wb}\left(a_{wb},b_{wb}\right)=\left(\frac{b_{wb}}{a_{wb}}\right)\cdot\left(\frac{x}{a_{wb}}\right)^{b_{wb}-1}\cdot\exp\left(\left(\frac{-x}{a_{wb}}\right)^{b_{wb}}\right) \quad (12)$$

with a_{wb} as scale parameter and b_{wb} as parameter of shape [Verma 2014]. The bimodal approach with two Weibull distributions is:

$$F\left(a_{wb1},b_{wb1},a_{wb2},b_{wb2}\right)= q\cdot p_{wb1}\left(a_{wb1},b_{wb1}\right)+(1-q)\cdot p_{wb2}\left(a_{wb2},b_{wb2}\right) \quad (13)$$

The residuals R_{chy} and R_{wb} were determined analogously to section 4 and are listed in Tab. 1. It is obvious that both alternative methods describe the available data well but worse than the approach using two normal distributions.

6 CONCLUSION AND OUTLOOK

The presented investigations and results show, that protection times of laser safety barriers are not normal distributed as supposed in the relevant standard IEC 60825-4}. Instead, they show a bimodal behaviour, which can be described using a bimodal approach of the sum of two weighted normal distributions. Utilising spline interpolation and kernel density estimation, the empirically obtained values can be approximated very well with this method. Alternative approaches with a normal and a Cauchy respectively with two Weibull distributions were tested but described the data worse than the first approach.

A bimodal distribution means, that there are two populations with unknown origin. To develop a reliable method for the determination of safe protection times, knowledge of the reason for this has to be gained. This can be done by tests of influences of sample surface, sample size, deviations within the material, fluctuations of the laser output power, a focal shift and much more. Finally it is to state, that the common method for determining protection times should thought over recognising these results.

7 APPENDIX

7.1 Script for spline interpolation

```
counts_wz_spline=[0,counts_wz_rel,0];

delta_wz_spline=centers_wz_rel(2)-centers_wz_rel(1);
centers_wz_spline_Anf=centers_wz_rel(1)-delta_wz_spline;
centers_wz_spline_End=centers_wz_rel(end)+delta_wz_spline;
centers_wz_spline=[centers_wz_spline_Anf,centers_wz_rel,cen
ters_wz_spline_End];
xq=(min(x)-s_emp):0.01:(max(x)+s_emp);
vq=interp1(centers_wz_spline,counts_wz_spline,xq,'spline');
for i=1:1:length(vq)
if vq(i)<0
  vq(i)=0;
end
end

[pks,locpeaks]=findpeaks(vq);

Array_peaks=zeros(20,10);
for i=1:1:length(pks)
Array_peaks(i,1)=locpeaks(i);
Array_peaks(i,2)=pks(i);
end

Array_peaks = sortrows(Array_peaks, -2);

cell_test_norm=cell(100,20);
x_ber=-8:0.01:8;
mu=0;
vec_sigma=(0.05:0.005:1);
```

```
for i=1:1:length(vec_sigma)
cell_test_norm{i,1}=vec_sigma(i);
end

for i=1:1:length(vec_sigma)
cell_test_norm{i,2}=normpdf(x_ber,mu,vec_sigma(i));
end

for i=1:1:length(vec_sigma)
cell_test_norm{i,3}=diff(normpdf(x_ber,mu,vec_sigma(i)));
end

for i=1:1:length(vec_sigma)
cell_test_norm{i,4}=cell_test_norm{i,3}(525);
end

vec_diff_vq=diff(vq);
for i=1:1:length(locpeaks)
ind_15=Array_peaks(i,1)+25;
Array_peaks(i,3)=vec_diff_vq(ind_15);

for i=1:1:length(vec_sigma)
cell_test_norm{i,6}=diff(diff(normpdf(x_ber,mu,vec_sigma(i))));
end

vec_hilf_norm_max=cell_test_norm{:,2};
[r,t]=max(vec_hilf_norm_max);
for i=1:1:length(vec_sigma)
cell_test_norm{i,7}=cell_test_norm{i,6}(t);
end

vec_ddiff_vq=diff(diff(vq));
for i=1:1:length(locpeaks)
ind_max=Array_peaks(i,1);
Array_peaks(i,6)=vec_ddiff_vq(ind_max);
end

for i=1:1:length(vec_sigma)
for k=1:1:length(locpeaks)
cell_test_norm{i,8+k}=cell_test_norm{i,4}-Array_peaks(k,3);
end
end

for k=1:1:length(locpeaks)
vec_hilf_min=zeros(50,1);
 for i=1:1:length(vec_sigma)
   vec_hilf_min(i)=cell_test_norm{i,8+k};
 end
   vec_hilf_min=vec_hilf_min(vec_hilf_min~=0);

 [val_min,ind_min]=min(abs(vec_hilf_min));
sigma_schaetz=cell_test_norm{ind_min,1};
Array_peaks(k,4)=sigma_schaetz;
end

for i=1:1:length(vec_sigma)
for k=1:1:length(locpeaks)
cell_test_norm{i,15+k}=cell_test_norm{i,7}-Array_peaks(k,6);
end
end

for k=1:1:length(locpeaks)
vec_hilf_min=zeros(100,1);
 for i=1:1:length(vec_sigma)
   vec_hilf_min(i)=cell_test_norm{i,15+k};
 end
   vec_hilf_min=vec_hilf_min(vec_hilf_min~=0);
 [val_min,ind_min]=min(abs(vec_hilf_min));
sigma_schaetz_kr=cell_test_norm{ind_min,1};

Array_peaks(k,7)=sigma_schaetz_kr;

end

if length(locpeaks)>1
z=Array_peaks(1,2)/Array_peaks(2,2);
hv_bi_nv=z/(1+z);
else
hv_bi_nv=1;
end

if(length(locpeaks)>1)
mu1=xq(Array_peaks(1,1));
sigma1=Array_peaks(1,7);

mu2=xq(Array_peaks(2,1));
sigma2=Array_peaks(2,7);
elseif(length(locpeaks)==1)
mu1=xq(Array_peaks(1,1));
sigma1=Array_peaks(1,7);
mu2=0;
sigma2=1;
end
```

7.2 Script for kernel density optimization

```
mw_emp=mean(data);
s_emp=std(data);

x=(data-mw_emp)/s_emp;
x=sort(x);

vec_test_ks=ksdensity(x,x_ber);

fun_test_norm=@(param,x_ber)
param(1)*normpdf(x_ber,param(2),
param(3)) + (1-param(1))*normpdf(x_ber,param(4),param(5));

param_guess=[hv_bi_nv,mu1,sigma1,mu2,sigma2];

[param_opt,resnorm_bimodal_opt]=lsqcurvefit(fun_test_norm,
param_guess,x_ber,vec_test_ks);
```

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CONTACTS

M. Sc. Dipl.-Ing. (FH) Florian P. Lugauer
M. Sc. Fabian Wimmer
Prof. Dr.-Ing. Michael F. Zaeh
iwb – Institute for Machine Tools and Industrial Management
Technical University Munich
Boltzmannstr.15, Garching, 85748, Bavaria/Germany
Tel.: +49 89 289 15554
e-mail: florian.lugauer@iwb.tum.de
www.iwb.tum.de