# POLYNOMIAL INTERPOLATION IN 5-AXIS MILLING

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This study is an introduction to the five-axis milling using polynomial interpolation approach. Progressive milling technologies using different spline interpolations in the control system Sinumerik 840D were compared for a defined number and distributions of the controls points of a planar curve. A tool trajectory for the five-axis CNC milling of a concave profile was generated as the superposition of the third order polynomials, that had been generated with a special algorithm in *Mathematica*.

**KEYWORDS**

polynomial interpolation, spline, control system, control point, milling

## INTRODUCTION

Five-axis milling of the complex shapes is one of the most demanding and the most effective machining operation in term of the machining time. Very complex components can be machined in one operation without additional re-clamping [Méry 1997].

Five-axis machines have to provide the accurate axis positioning, precise axis acceleration and deceleration and high rigidity during machining. Machines for high speed cutting (HSC) and high feed cutting (HFC) with their lighter mass allow achieve higher cutting speeds, feed speeds and higher acceleration rates compared to standard machines. However, application of high speed cutting or high feed cutting can result in self-exited vibrations that generate discontinuities in the tool path trajectory affect surface quality [Altintas 2014].

A design of the appropriate tool path trajectory for the five-axis machining is the basic prerequisite for the accuracy and quality of the final complex shape of the workpiece. Types of the trajectories are basically divided in to the linear interpolation, circular interpolation and different spline interpolations [Altintas 2014]. All types of the milling strategies prevail with some advantages and disadvantages. However, they differ in the continuity of the tool path and the ability to pass through control points defining machined surface fundamentally [Siemens manual 2004].

## 1 THE BASIC ALGORITHM FOR GENERATING OF A POLYNOMIAL INTERPOLATIONS IN CNC MILLING

### 1.1 *Toolpath generation*

Toolpath generation has a direct impact on the surface quality of the machined component. Various surface shapes can be machined using the linear interpolation, circular interpolation or different spline interpolations (A-spline, B-spline, C-spline, NURBS). The linear interpolation, despite of its discontinuity in the curvature, is the most frequently applied strategy because of its easy to use [Msaddek 2014]. Depending on the curvature and complexity of the surface, the long path segments are divided into shorter discrete segments (linear or polynomial). The resulting toolpaths are generated by a CAM software and transformed through a post processor into a work coordinate system of the machine. It is necessary to ensure that generated program contains information about the positioning of the tool tip, directional and normal vector of the tool axis and the tangential infeed for each toolpath segment. The generation of the reference motion commands for individual axes is based on tool path planning, tangent infeed profile and a real position interpolation within servo control loops. The CAM system generates short segments from initial long spline paths. These short segments are preceded to CNC as linear toolpaths. If the system contains a real NURB interpolator (Non-Uniform Rational B-spline) then splines can be interpolated [Altintas 2014]. The difference between these two methodologies is notable - the linear interpolation leads to discontinuous velocities, decelerations and accelerations at the nodes points. On the other hand, NURBS interpolator allows the smooth continuous tool paths based on the parameterized spline curves [Siemens manual 2005].

Some recent studies have examined an influence of the interpolation type on the complex shapes of the machined surfaces. Helleno and Schützer [Helleno 2006] examined the limits of the linear interpolations during machining of the moulds and presented benefits that can be related to the use of the spline interpolations. Meng-Shiun Tsai et al. [Tsai 2009] developed a new look-ahead algorithm with spline-fitting interpolation scheme which consists of the spline-fitting and acceleration and deceleration feed rate planning. Based on the methodology the conditions to ensure the continuity of the position, slope and curvature at each nodal point of the polynomial curve or linear segment are derived. Zhang et al. [Zhang 2011] introduced the parametrical cubic spline curve transitional approach to improve the machining efficiency within the permissible machining accuracy range for the high-speed machining. Langeron et al. [Langeron 2004] provided a unique format for the computation of 5-axis toolpaths using the B-spline curves. Pateloup [Pateloup 2010] presented the B-spline approximation of circle arcs and straight lines for pocket machining. As it is a very actual problematic many other studies have been published on this topic in the recent years.

### 1.2 *Types of the spline interpolation*

Splines belong to a group of the effective tool path functions allowing smooth and complex shapes. Spline curves consist of implicit equations or parametric functions. However, most of CAM systems use parametric forms of spline curves because of the practical control of the machine axis motions [Sencer 2003]. Basic spline curves which are used in the machine control system are A-spline (Akima spline), B-spline (Bezier spline), C-spline (Cubic spline) and in some cases also general polynomial interpolation. However, the use of mentioned splines may not be the same for all control systems.

***A-spline***

A-spline directly passes through the node points of the curve but is not continuous in the curvature. A-spline interpolation is based on the polynomials of the third degree and given boundary conditions as showed in the equation (1) [Siemens manual 2004].

$$s(x) = a_0 + a_1(x - x_i) + a_2(x - x_i)^2 + a_3(x - x_i)^3$$
$$where\ x_i \le x \le x_{i+1} \quad (1)$$

***B-spline***

Although the spline is called B-spline, it is actually NURBS interpolation. B-spline curves are basically free-form curves that consist of the segments expressed by Bezier curves, which are connected to each other with the highest degree of the continuity (the continuous first and second derivatives at the nodal points at least) [Msaddek 2014]. B-spline are determined by n+1 control points and by the degree p, which determines the degree of the individual arcs of the curve. B-spline does not pass directly through the nodal points of the control polygon, but only approaching them according to the "weight" specified in the control system [Siemens manual 2005]. The B-spline is always tangential to the control polygon at the start and end points and does not generate undesirable vibrations [Siemens manual 2004].

***C-spline***

The C-spline is an interpolation by a cubic polynomial. C-spline passes directly through the nodal points of the curve and it is continuous in the curvature that means it has a low curvature variation [Msaddek 2014]. However, the C-spline is characterized by a high tendency to the oscillation. This spline is mainly used when nodal points belong to the analytically known curve [Siemens manual 2004].

***Polynomial interpolation***

Polynomial interpolation is often mistakenly interchanged with spline interpolation, which also uses third-order polynomials, but in this case, it is not a kind of spline interpolation. In CNC milling the polynomial interpolation is mainly the interface for the programming of the externally created spline curves. This type of the interpolation can be used when curves or complex shapes are described only in the mathematical software such as MATLAB or *Mathematica* without need of the CAD models. Spline segments of the mathematically described curves can be programmed directly without need of the CNC system to compute polygon coefficients [Siemens manual 2004].

The most commonly used is the third-degree polynomials but fifth degree polynomials can be eventually used if allowed by the control system.

## 2 EXPERIMENTAL VERIFICATION

### 2.1 *Spline interpolation and polynomial interpolation in 2D*

In order to compare different spline interpolations with linear interpolation and polynomial interpolation, the 17 control points of the analytically known curve were defined (see Table 1).

| Point | X [mm] | Y [mm] | Point | X [mm] | Y [mm] |
|---|---|---|---|---|---|
| **1** | 0 | 1 | **10** | 180 | 7 |
| **2** | 20 | 6 | **11** | 200 | 9 |
| **3** | 40 | 12 | **12** | 220 | 11 |
| **4** | 60 | 6 | **13** | 240 | 8 |
| **5** | 80 | 2 | **14** | 260 | 2 |
| **6** | 100 | 6 | **15** | 280 | 13 |
| **7** | 120 | 12 | **16** | 300 | 2 |
| **8** | 140 | 6 | **17** | 320 | 2 |
| **9** | 160 | 1 | | | |

**Table 1.** Defined control points of the curve

Four CNC programs for A-spline, B-spline, C-spline and linear interpolation were generated in the control system SINUMERIK 840D and machined using five-axis machining centre MCV 1210 (whole carbide milling cutter FRAISA HM MG10 ⌀20mm, $v_c$=200 m/min, $v_f$=400 mm/min, $a_p$=15 mm and $a_e$=2 mm - – according to the previous experimental studies) – see Figure1.

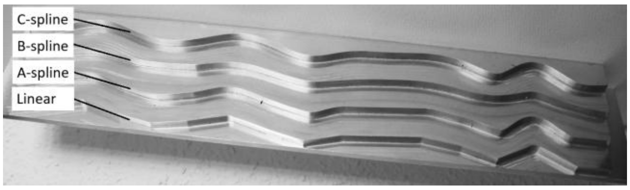


**Figure 1.**Machined curves using spline and linear interpolations

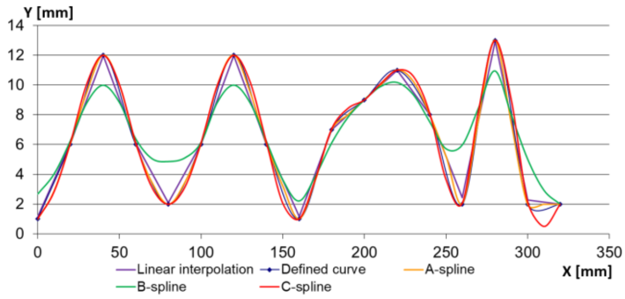


**Figure 2.** Comparison of spline interpolations

For the conditions when a very limited number of the control points is defined, the use of the B-spline is the least desirable interpolation approach, because of their passing off. In this case even if the weight on point is set to be highest, the B-spline is not able to keep the desired curve. The most appropriate solution when curve is defined by a small number of points is the A-spline and eventually C-spline – see Figure 2 (measured by MarVision MM 420). Surface topography was similar for all types of interpolation with Sa values between 1,3 µm to 2 to µm (measured by ALICONA IF-G4).

In order to reproduce the same curve using a polynomial interpolation approach, it is necessary to define the parametric polynomials of the third degree as x(t) and y(t). These parametric polynomials were defined by the cubic Bezier curves with four main control points $P_0$, $P_1$, $P_2$ and $P_3$ and working on the approximate principle - see equation (2) [Favrolles 1998]. The cubic Bezier curve passes through the initial and end point $P_0$ and $P_3$. Points $P_1$ and $P_2$ indicates only the shape of the curve – see Figure 3.

$$C(t) = P_0(1-t)^3 + 3P_1 t(1-t)^2 + 3P_2 t^2(1-t) + P_3 t^3$$
$$where\ 0 \le t \le 1 \qquad (2)$$

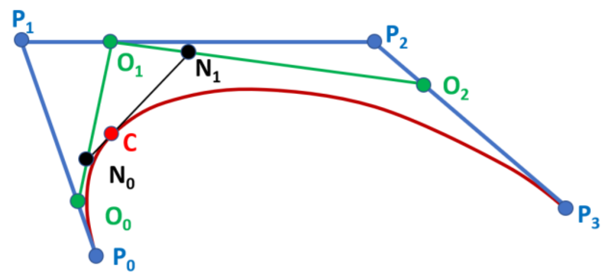


**Figure 3.** The cubic Bezier curve [Favrolles 1998]

The analysed curve defined by 17 points was divided into 8 sections with 3 points per each. The control points of the Bezier curves of each section were defined in software *Mathematica 8.0.1*. Initial and end point of each section were identical to the control points of the curve, curve passes directly through them to ensure continuity of the curvature. Remaining two points of the cubic Bezier curves were defined in order to guarantee that the curve is closely approaching the middle control point of each section. The *Mathematica* was used later to generate polynomials of the third degree for x(t) and y(t) - see Table 2.

| | |
|---|---|
| section 1 | $x(t)= 73.5t - 94.4539t^2 + 60.9539t^3$ |
| | $y(t)= 1 + 33t^2 - 22t^3$ |
| secion 2 | $x(t)= 40 + 60t - 74.539t^2 + 54.539t^3$ |
| | $y(t)= 12 - 3t^2 + 20t^3$ |
| section 3 | $x(t)= 80 + 60t - 40.956t^2 + 20.956t^3$ |
| | $y(t)= 2 + 30t^2-20t^3$ |
| section 4 | $x(t)= 120 + 160t - 360t^2 + 240t^3$ |
| | $y(t)= 12 - 37t^2 + 26 t^3$ |
| section 5 | $x(t)= 160 + 59.981t - 211.905t^2 + 191.924t^3$ |
| | $y(t)= 1 + 1.49953t - 0.19335t^2 + 6.69382t^3$ |
| section 6 | $x(t)= 200 + 61.538t - 64.614t^2 + 43.076t^3$ |
| | $y(t)= 9 + 6.1538t - 1.4615t^2 - 5.6923t^3$ |
| section 7 | $x(t)= 240 + 219.512t - 340.024t^2 + 160.512t^3$ |
| | $y(t)= 8 - 49.3902t + 113.78t^2 - 59.3902t^3$ |
| section 8 | $x(t)= 280 + 150t - 330t^2 + 220t^3$ |
| | $y(t)= 13 - 33t^2 + 22t^3$ |

**Table 2.** Third degree polynomials for curve sections

Based on the polynomials coefficients for the axis X and axis Y, a program was made (see Figure 4) and machining was performed at the MCV 1210/Sinumerik 840D with cutting conditions $v_c$=14 m/min, $a_p$=1 mm, $v_f$=300 mm/min, $a_e$=3 mm, using all-carbide cutter FRAISA HM MG10 ⌀3mm (according to the previous experimental studies) – see Figure 5.

```
POLY
PO[X]=(40,-94.4539,60.9539) PO[Y]=(12,33,-22) PO[Z]=(-10,0.0,0.0)
PO[X]=(80,-74.539,54.539) PO[Y]=(2,-30,20) PO[Z]=(-10,0.0,0.0)
PO[X]=(120,-40.956,20.956) PO[Y]=(12,30,-20) PO[Z]=(-10,0.0,0.0)
PO[X]=(160,-360,240) PO[Y]=(1,-37,26) PO[Z]=(-10,0.0,0.0)
PO[X]=(200,-211.905,191.924)PO[Y]=(9,-0.19335,6.69382) PO[Z]=(-10,0.0,0.0)
PO[X]=(240,-64.614,43.076) PO[Y]=(8,-1.4615,-5.6923) PO[Z]=(-10,0.0,0.0)
PO[X]=(280,-340.024,160.512)PO[Y]=(13,113.78,-59.3902) PO[Z]=(-10,0.0,0.0)
PO[X]=(320,-330,220) PO[Y]=(2,-33,22) PO[Z]=(-10,0.0,0.0)
```

**Figure 4.** POLY program in Sinumerik 840D

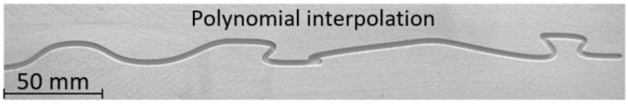


**Figure 5.** Machined curve using polynomial interpolation

For the section 1, section 2, and section 3, the approximation using Bezier curve was close to the desired curve. However, in the next section the limited number of the defined points started to affect the shape of the curve. In these sections, the loops and reciprocal tangents on the discontinuities of the sections started to appear. In the area of the superposition of each section, it was not possible to modify direction of the tangent in order to keep a continuity of the curvature correctly. However, for the simple curvature that is defined by the controls points spaced less than 3 mm from each other, this algorithm seems to be the ideal solution for the desired curve.

### *2.2 Polynomial interpolation in five-axis*

Polynomial interpolation in five-axis milling was performed on the plane 30x30 mm. Using a linear combination of Bernstein base polynomials of the degree *n* that form the base vector space of the same degree [Favrolles 1998], the plane of the desired shape was developed – see Figure 6.

For a standard face milling operation using ball end mill where the tool is positioned perpendicularly to the machined surface, the cutting speed in the axis of the tools goes to zero. This leads not to the cutting of the material but simple forming of the material by the middle of the tool. So, it is recommended to keep the defined inclination angle of the tool while milling.

Tool inclination can be performed in two directions. First, tool tilting relative to the normal of the surface in the feed direction ($\beta_f$) and tool tilting in the direction perpendicular to the feed ($\beta_n$) [Sadilek 2006].

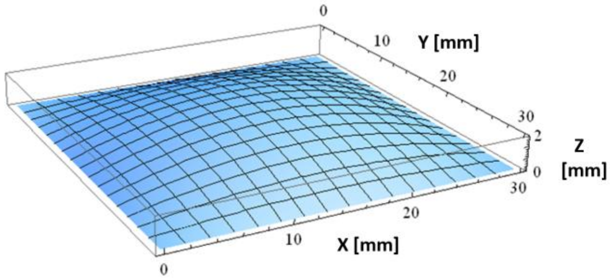


**Figure 6.** Surface generated by Bernstein polynomials

***Polynomial interpolation in continuous five-axis milling***

The continuous five axis machining involves translation axes and rotational axes in to the simultaneous movement.

In order to use polynomial interpolation approach, the approach with the tool axis identical to the surface normal ($\omega_f$=0°, $\omega_n$=0°) was chosen despite its disadvantages mentioned previously. The third order polynomials were generated directly for the zero point of the tool – Figure 7.

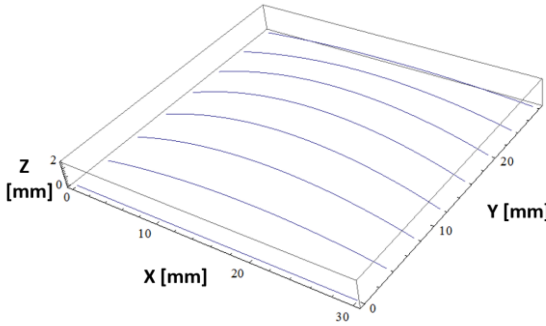


**Figure 7.** Toolpath polynomials

The third order polynomials can be defined for the individual tool paths. The program made with *Mathematica* offers a post-script for post-processor functions and basic algorithm of CNC programming of polynomials for the Heidenhain iTNC530 control system also. Based on the polynomial interpolations for rotational axis B, C and all translation axes can be generated.

The experimental milling using the polynomial interpolation approach was performed with the milling center MIKRON HSM600U/Heidenhain iTNC 530 and use of the ball end mill Sandvik R216.62-12030-AO13G 1010 ⌀12 mm ($v_c$=263 m/min, $v_f$=1680 mm/min, $a_p$=0,2 mm and $a_e$=0,3 mm – according to the previous experimental studes) – Figure 8.

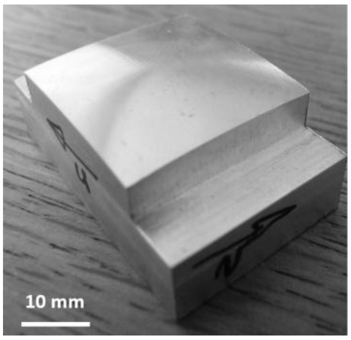


**Figure 8.** Machined surface using the polynomial approach

***Polynomial interpolation with fixed angle of the tool inclination***

In order to optimize cutting conditions and avoid the non-cutting phenomenon in the axis of the tool the mathematical algorithm based on the definition of the ball end mill, the number of passes and the inclination of the tool (in two orthogonal planes with $\omega_f$=30° and $\omega_n$=10° angles) has been made with the mathematical software *Mathematica*. Program allowed generate third order polynomials for the individual tool paths. The polynomials referred to the contact points of the tool and the workpiece – see Figure 7.

Consequently, the points of the contact had to be transformed to the tool zero point on the tool axis. This was achieved by a coordinate transformation and by inverse kinematics, which transformed the coordinate system of the workpiece into the coordinate system of the machine – see Figure 9.

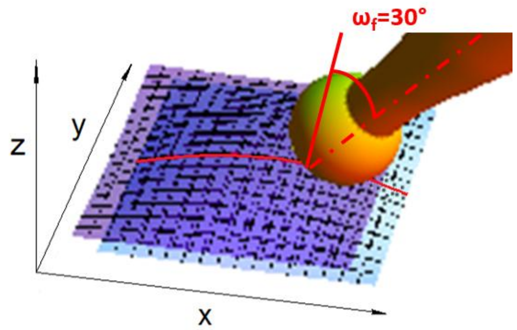


**Figure 9.** Positioning of the ball end mill (in the first plane)

However, during this transformation from the contact point to the zero point at the tool axis, the third-degree polynomials were in some cases transformed to the rational functions which cannot be processed by the CNC control systems. In case of the polynomial interpolation with fixed angle of the tool inclination is necessary to make a special tool correction in order to get the right tool positions and trajectory.

## 3 CONCLUSIONS

Nowadays, most of all modern control systems contain function allowing external polynomial programming (Sinumerik, Heidenhain, Fanuc, Mazatrol, etc.) but this application is not widely used in the serial production. The main reason is the need for an external mathematical calculations and programming for the tool paths. However, direct polynomial interpolation approach can be used when current CAD/CAM programming is not able to reach path specifications appropriately. Polynomial interpolation approach can, in some specific cases, substitute also the phase of reverse engineering where surface of the workpiece can be described mathematically, based on the optimized number of the control points. However, a very good surface quality can be achieved when a convenient machine set-up, optimal algorithms, modern tooling and cutting conditions are used.


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