STRUCTURAL DESIGN OF NON-STANDARD GEAR TRANSMISSION

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DOI: 10.17973/MMSJ.2024_11_2023146

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Non-standard gear transmissions are increasingly finding their applications in practice, primarily being designed for various applications. While standard circular gears are designed for a constant gear ratio, non-standard gears have a variable gear ratio that changes. This paper addresses the issue of designing non-circular gears for a specific range of gear ratios. An elliptical shape for the gears was chosen. The uniqueness of the proposed solution lies in selecting an ellipse as the evolute for creating the involute curve of the gear tooth.

KEYWORDS

Non-standard gear, non-circular gear, design, elliptical gear

1 INTRODUCTION

Gears are among the most widely used transmission mechanisms [Puskar 2024]. They are fundamental components in machines, enabling the transmission and transformation of mechanical energy and motion [Kuczaj 2023]. The basic part of a gear transmission is a gear pair consisting of a driving and a driven gear [Drewniak 2023]. It can be said that gears have become a symbol of mechanical engineering.

Standard gears, characterized by a constant gear ratio, are the most commonly used in practice. This means that as the driving gear rotates, the driven gear rotates uniformly, maintaining a constant gear ratio throughout one revolution [Malakova 2021]. The teeth of these standard gears have the same shape and symmetrical profiles (or asymmetrical in rare cases) [Saga 2019]. These standard gears are designed to transmit torque with minimal noise and wear and maximum efficiency [Duhancik 2024].

Non-standard gears, which do not have a constant gear ratio throughout one revolution, also find their applications in practice [Juzek 2017]. These gears are designed for specific applications to achieve variable gear ratios, axle lifts, oscillations, and other essential properties and characteristics [Bratan 2023]. This topic is gaining more attention. Mostly, these involve non-circular gears (NCG).

Research in this area focuses on designing the active curve shape. Biing, BW [Biing 2009] worked on designing oval centrodes with convex curves using curvature analysis. A singular point equation was used for undercutting analysis. Additionally, a computer simulation program has been developed to generate tooth profiles of oval gears with circular arc teeth having convex centers without undercuts. Various numerical examples illustrate the computer design process.

Zhou [Zhou 2020] focused on designing the tooth profile of non-circular gears with a variable gear ratio for a transplanting mechanism. Based on the gear shaping principle using gear-cutting tools, a theoretical model for calculating the tooth profile of non-circular gears with additional adjustments was derived.

Yang [Yang 2023] also dealt with generation the tooth profile of non-circular gears. This article created a mathematical model, derived the equation of its axode, and proposed a new method for generating the tooth profile using relative velocity.

Liu [Liu 2022] proposed a new variable involute and an incomplete variable cycloidal composite tooth profile and constructed a conjugate gear model based on the envelope method.

The quality of gears is predominantly determined by their geometric design. Even the best materials cannot ensure reliability if the geometric design is flawed. Conversely, an excellent geometric design can sometimes save costs on expensive materials. This paper describes the procedure for creating a geometric model of a non-standard elliptical gear transmission with eccentrically placed gears and a smoothly changing gear ratio for specific parameters.

2 DEVELOPMENT AND APPLICATION OF NON-STANDARD GEARS

Humanity has known and used gears since before the common era, primarily in water mills, water pumping, or lifting heavy loads. This is evidenced by Aristotle (384 - 322 BC), who understood gear transmissions. Mathematician and physicist Archimedes (287 - 212 BC) used gears for water pumping (Archimedes' screw from 278 BC). Some of the oldest gear mechanisms, part of the first mechanisms, include the planetary gear remnants from the first century BC found near the island of Antikythera.

Giovanni de Dondi (1330–1388) was among the first to use non-standard gears in his astronomical device, known as the Astrarium, which simulated the solar system. He also authored the manuscript "Tractatus Astrarii," documenting the design and manufacture of sophisticated clocks. This manuscript includes one of the first documented methods for designing non-circular gears [Angeles 2004].

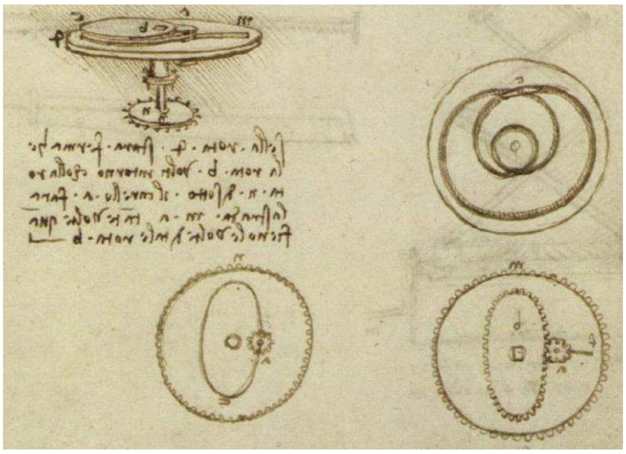

Figure 1. The sketches made by Leonardo da Vinci in the collection of "Codex Madridii" [Moon 2007]

Many gears used today were found in Leonardo da Vinci's (1452 - 1519) schemes. His work referenced non-circular gears, including a clock and an elliptical mechanism. Such gears, with variable gear ratios, were ideal for this purpose (Fig. 2) [Moon 2007].

Franz Reuleaux (1829–1905) found several new applications for non-circular gears in industry, proposing their use in textile machines, mechanical presses, and high-torque hydraulic motors as alternatives to cam mechanisms [Pickover 2009].

Figure 2. A historical model of a non-circular gear [Pickover 2009]

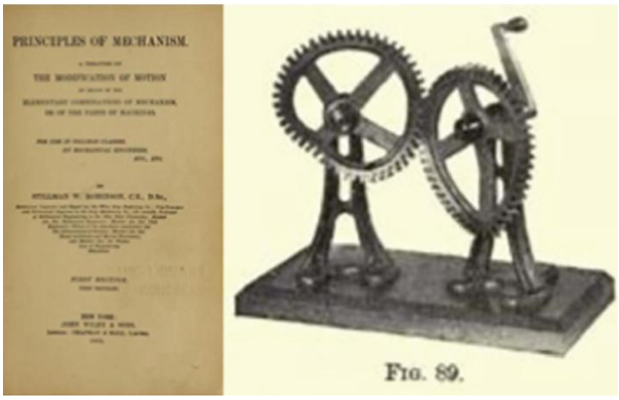

Figure 3. Sample from the book "Principles of Mechanism" by S.W. Robinson [Robinson 2010]

S.W. Robinson (1838–1910) detailed the basic principles of non-circular gears' construction and operation in "Principles of Mechanism" (Fig. 3). His work includes calculations for the tooth profile and gear ratio, as well as applications with examples and photographs of various non-circular gears used in engineering [Robinson 2010].

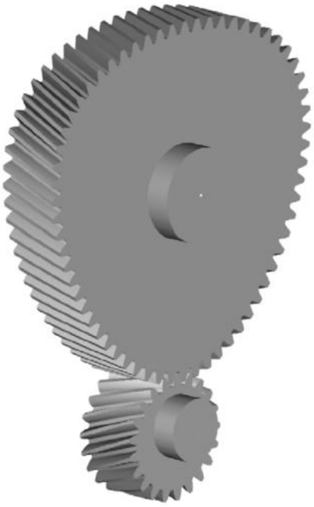

Figure 4. Eccentric involute non-circular gearing [Litvin 2009]

The development of these non-standard mechanisms advanced in the 20th century. F.L. Litvin (1914-2017) focused on designing and manufacturing the main types of non-circular gears (Fig. 4). His work analyzed conventional and modified elliptical gears, eccentric gears, oval gears, gears with protrusions, and twisted gears. He also offered new developments to expand the application of non-circular gears for speed variation and function generation [Litvin 2009].

Artobolevsky (1905-1977) [Litvin 2008] described several mechanisms composed of non-circular gears. His works demonstrated the functioning of these mechanisms, detailing their structures and properties. He also showed that the involute profile is not the only option but is the most successful for current technology.

In the past, high manufacturing costs limited the use of non-circular gears [Kuric 2022]. Today, thanks to modern technologies and computer-controlled machining, as well as new mathematical models for calculating non-circular shapes, non-standard gears were actively replacing cam mechanisms [Moravec 2021]. Specially designed devices, rapid prototyping, and EDM machining have made it possible to produce non-circular gears through duplication or rolling [Olejarova 2021]. The increased design and manufacturing potential of non-circular gears explains the growing interest in this type of gear.

Applications of non-circular gears include:

- Textile machines for improving kinematics, optimizing the process.
- Window shading panel drives to introduce vibrations that disrupt natural oscillations.
- Mechanical presses for optimizing the work cycle kinematics.
- High-torque hydraulic motors for partitioned drives.
- High-performance starters, providing progressive torque for easier machine starting, helping to overcome starting inertia.
- Forging machines to optimize work cycle parameters (reducing pressure dwell time).
- Automotive industry applications, such as diesel engines and more.

3 ELLIPTICAL GEARING ECCENTRICALLY STORED WITH A CONTINUOUSLY CHANGING TRANSMISSION NUMBER

Based on practical requirements, a geometric model of a gear transmission with a smoothly changing gear ratio was needed to meet the conditions of proper meshing. The gear transmission was designed for specific parameters, comprising two identical gears. The harmonically repeating gear ratio ranges from 0.5 through 1.0 to 2.0 and back within one rotation of the meshing gears. The gears have 24 teeth each, where $z_1 = z_2 = 24$, with a normalized module value of $m_n = 3.75$ mm. The axial distance between the gears is $a = 90$ mm, designed for a single rotation direction.

The task provider also supplied a rough model of the gear pair for illustration (Fig. 5), which did not meet the conditions of proper meshing.

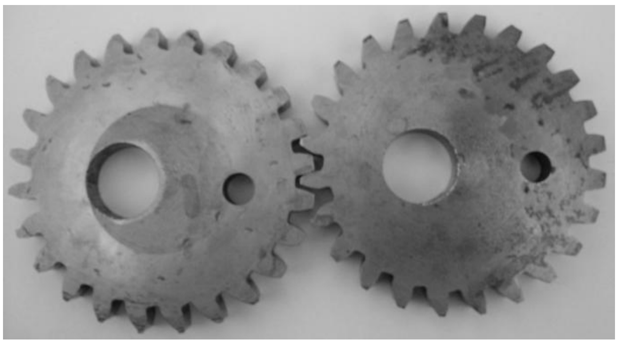

Figure 5. Model of the delivered gear pair

The geometric design of non-standard gears largely determines the quality of the gear transmission. A flawed geometric design will not ensure reliability. Conversely, a good geometric design can sometimes save costs on expensive materials.

The first step in the design of the gearing is the design of the pitch curve. In "standard" gears, the pitch curve is formed by a pitch circle with the axis of rotation coincident with the center of the pitch circle. For non-standard gears, it is not a circle.

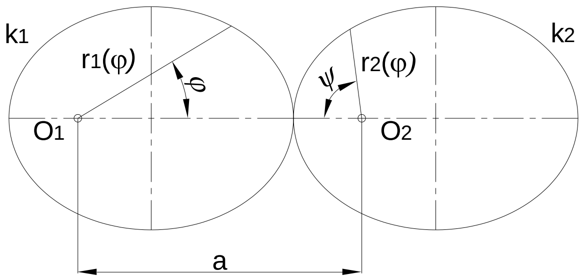

Figure 6. Non-circular pitch curves

There are two rollers rolling together without slippage in Figure 6, which represents non-circular gears, provided that there is no addendum modification, and the nominal axle distance is used. Pitch curves are divided in z parts that are p in length, where z is the gear's number of teeth, and p is the pitch. The gears are represented by the two pitch curves k_1 and k_2 with centres in the points O_1 and O_2. The variable pitch curve radii – $r_1(\varphi)$ and $r_2(\varphi)$ of the non-circular gears are determined by the required course of the transmission ratio i for the kinematics of the ram:

$$i(\varphi)=\frac{d\varphi}{d\psi}=\frac{1}{\psi'(\varphi)}=-\frac{r_2(\varphi)}{r_1(\varphi)}=\frac{\omega_1(\varphi)}{\omega_2(\varphi)} \tag{1}$$

where $\omega_1(\varphi)$ and $\omega_2(\varphi)$ are the angular velocity functions for the gear 1 and 2, respectively.

With a given constant centre distance

$$a=constant=r_1(\varphi)+r_2(\varphi) \tag{2}$$

the transmission function $\psi'(\varphi)$ describes the relation between the pitch curves of the non-circular gears. In the case of an aperiodic non-circular gears, angular positions of the members are limited. Usually, a specific ratio function is used, for example a logarithmic function.

Based on the results of the problem, an elliptical shape of the gears was chosen. The geometric center of the gear is not the center of rotation of the gear. The center of rotation of the gear is chosen at the focus of the ellipse. The dimensions of the selected pitch ellipse of the gear are shown in Fig.7.

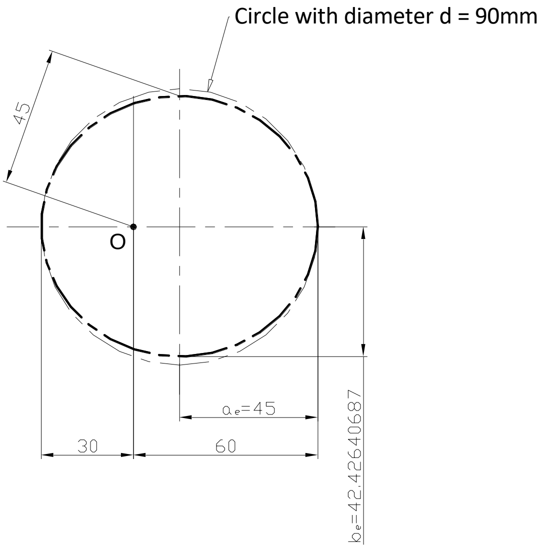

Figure 7. Dimensions of the designed pitch ellipse

The major axis of the pitch ellipse a_e = 45 mm was based on the given axial distance a = 90 mm. The rotation centers O_1 and O_2 were determined to create a gear with a time-varying gear ratio ranging from 0.5 through 1.0 to 2. The gear ratio of one whole, $r_1+r_2=90$ *mm*, where $r_1=r_2$ a $r_1/r_2 = 1.0$, was determined the size of the minor axis b_e = 42.42640687 mm (Fig. 7). Points O_1 and O_2 are also the foci of the pitch ellipse. It is based on the ellipse property that for any point on the ellipse, the sum of distances from points O_1 and the point plus the point and O_2 is always equal to twice the major axis, in this case, the axial distance.

Figure 8 shows the pitch ellipses of the co-occupying eccentrically mounted elliptical gears, as well as the division of the pitch ellipse according to the number of teeth (*z = 24*) into 24 length-matched pitches. These are two identical gears, and therefore the tooth designation is chosen to be the same on both co-occupying gears. For each pair of co-occupying teeth, the sum of the spacings e.g., $4O_1$ and $8O_2$ must be equal to the axial spacing (shown in the figure for the co-occupying teeth with the numerical designations 4 for the tooth of the driving wheel and 8 for the tooth of the driven wheel).

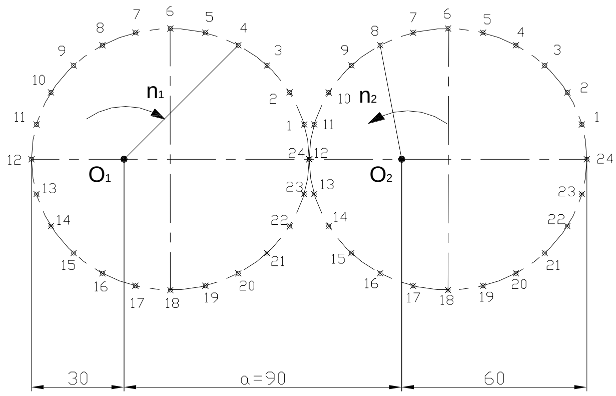

Figure 8. Pitch ellipses of the eccentrical gears

There are values of pitch radii in particular mesh points in Table 1. Their designation used symbols $r_{1\text{-}i}$ or $r_{2\text{-}j}$. The index 1 stands for the driving gear, index 2 for the driven gear, index i, or j is the order number of the meshing teeth (Fig. 8) at one revolution for the driving and driven gear.

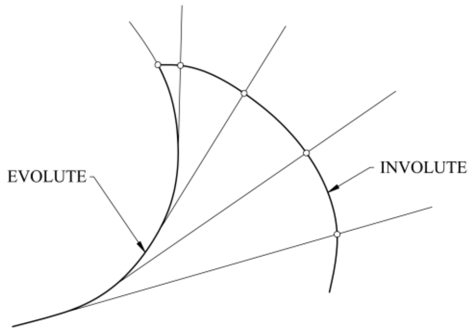

Figure 9. Involute, evolute scheme

The construction of the active tooth flank curve plays an important role. The tooth profile side curve consists of a cyclic curve, usually an involute, and a transition curve. Only the involute part of the tooth flank can act as the active part during meshing. The transition curve's task is to create a smooth transition between the involute part and the tooth root.

The involute generally arises as the path of a point during the unwrapping of a tangent line along a given curve, applying the arc length of the curve to its tangent line [22]. The geometric location of the involute curvature centers is called the evolute (Fig. 9).

Table 1 shows that variable gear ratio of the inter-meshing gears as well, in the range from *0.5* (the first pair of teeth mates, the tooth *24* of the driving gear is in mesh with the tooth *12* of the driven gear) through the gear ratio = *1.0* (if the tooth *6* of the driving gear is in mesh with the tooth *6* of the driven gear), up to the gear ratio = *2.0* (if the tooth *12* of the driving gear is in mesh with the tooth *24* of the driven gear) and back through the gear ratio = *1.0* to the gear ratio = *0.5.*

Table 1. Pitch radii and changing gear ratio

Tooth of driving gear – order number	Pitch radius $r_{1\text{-}i}$ (mm)	Tooth of driven gear – order number	Pitch radius $r_{2\text{-}j}$ (mm)	Gear ratio $u_i=r_2/r_1$
24	60	12	30	**0.5**
01	59.458972	11	30.541025	0.5136
02	57.891981	10	32.108009	0.5546
03	55.449504	09	34.550498	0.6230
04	52.336954	08	37.663046	0.7196
05	48.778898	07	41.221102	0.8450
06	45	06	45	**1**
07	41.221102	05	48.778898	1.1833
08	37.663046	04	52.336954	1.3896
09	34.550498	03	55.449504	1.6048
10	32.108009	02	57.891981	1.8030
11	30.541025	01	59.458972	1.9468
12	30	24	60	**2**
13	30.541025	23	59.458972	1.9468
14	32.108009	22	57.891981	1.8030
15	34.550498	21	55.449504	1.6048
16	37.663046	20	52.336954	1.3896
17	41.221102	19	48.778898	1.1833
18	45	18	45	**1**
19	48.778898	17	41.221102	0.8450
20	52.336954	16	37.663046	0.7196
21	55.449504	15	34.550498	0.6230
22	57.891981	14	32.108009	0.5546
23	59.458972	13	30.541025	0.5136

For standard circular involute gears, an involute with a circular evolute is used for tooth profiling. This principle was used for the first solution. The resulting shape of the gear created this way is shown in Fig. 10. The solution was incorrect due to tooth collisions.

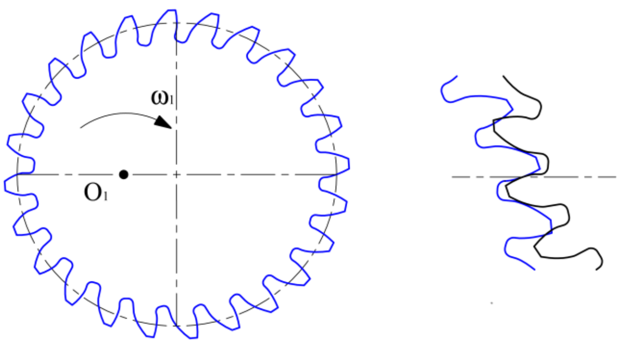

Figure 10. First solution, teeth collision

Based on this finding, more precise tooth construction was needed. The more precise construction assumed the evolute is an ellipse. In this solution, the first step to constructing the involute part of the tooth flanks is drawing tangents at the central contact points on the pitch ellipse. The involute construction was then carried out using a trochoid method. This method laboriously constructed an involute curve for each tooth separately for its left and right sides. This means the gear has asymmetrical teeth with different involute curves for the tooth flanks. The difference in constructing these tooth profiles is shown in Fig. 11. The methods used in references applied a basic circle as the evolute – shown on the left in the figure. The procedure described in this article is different, with an elliptical evolute. The difference in procedure is also visible in the tooth positions relative to the rotation center. A fundamental difference in the tooth shape results from different involute constructions.

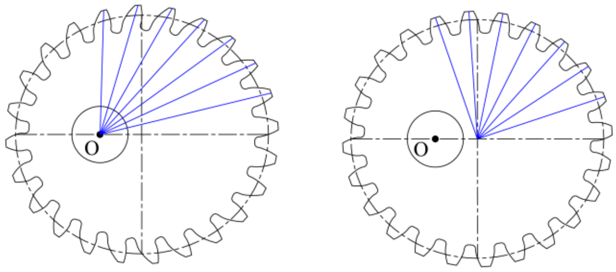

Figure 11. The shape of the gearing if the evolute is a) a circle, b) an ellipse

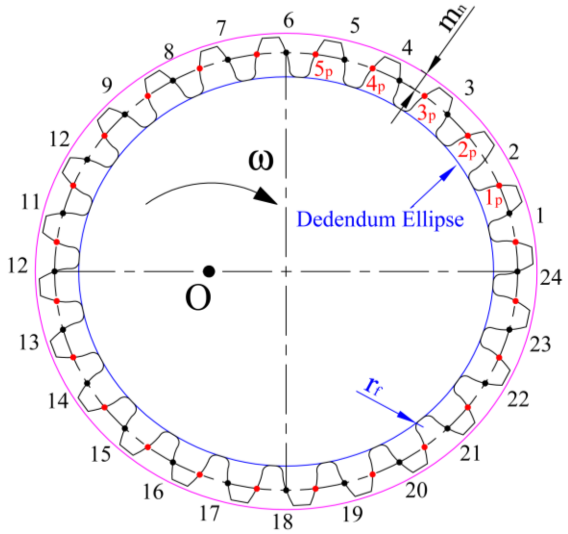

Figure 12. Resulting shape of the gearing

The resulting shape of the designed gearing is shown in Figure 12. The height of the tooth addendum is not equal to the modulus because there would be an involute meshing with the transition part. The head must be lowered so that proper meshing of the co-engaging gears occurs.

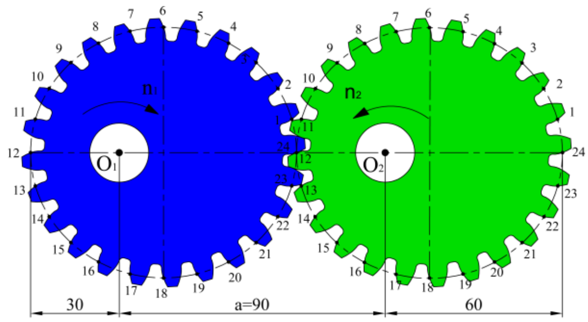

Figure 13. Elliptical gear model

The gears for the specified gearing with a time-varying gear ratio ranging from 0.5 through 1.0 to 2.0 were designed as elliptical, eccentrically mounted (Fig. 13), so that the conditions for correct meshing were met. The gear consists of a pair of identical gears with a tooth count of 24 and a standardised modulus value of m_n = 3.75 mm.

4 CONCLUSIONS

Gearboxes with a non-constant gear ratio, namely with a continuously variable gear ratio, are increasingly used in practice. The appropriate design of the pitch curve shape for the desired range of changing gear ratio is the first crucial step in successfully solving the problem.

An elliptical pitch curve shape was designed for the required continuously varying gear ratio. The gear train is formed by a pair of identical elliptical gears, with the centre of rotation at one of the foci of the pitch ellipse, i.e., they are eccentrically spaced gears. The active curve of the tooth profile is the involute, where unlike the involute of the tooth flank in "standard" circular gears, where the evolute of the involute is the base circle, and in this case the evolute of the involute is the ellipse. Each of the twelve teeth of a gear is different, the other twelve teeth of the same gear are the same. The flank curve is the involute and is different for the active and passive sides of the tooth. These are gears with an asymmetric tooth profile. The gear ratio for the proposed elliptical gear is not constant but varies continuously from 0.5 through 1.0 to 2.0 and back. Thus, the gear ratio varies within one revolution. A gear ratio value that is less than 1.0 means that it is a fast gear, a gear ratio value greater than 1.0 means that it is a slow gear.

The non-standard gearing designed in this way met all the requirements of proper meshing as well as the requirements placed on it by the thesis sponsor.

ACKNOWLEDGMENTS

The contribution was prepared within the solution of grant project KEGA 037TUKE-4/2024.

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