

# BASIC ALGORITHMS OF INPUT SHAPING AUTOTUNING

MACIEJ GNIADEK<sup>1</sup>, STEFAN BROCK<sup>2</sup>

<sup>1</sup> Poznan University of Technology, Institute of Control and Information Engineering and Amica Wronki S.A., Poznan, Poland

<sup>2</sup> Poznan University of Technology, Institute of Control and Information Engineering, Poznan, Poland

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e-mail: maciej.gniadek@amica.com.pl

Many mechanical systems contains flexible joints. The main problem in control of these kinds of objects are oscillations. The control strategy can be supplied by signal shaping methods. One of the simplest and most effective algorithm of reference signal shaping is named input shaping. The problem of input shaping tuning can be automatized. The paper shows two basic algorithms of input shaping autotuning. Every tuning method needs the information of object's natural oscillation frequencies and their damping. These information are extracted using object identification and Fast Fourier Transform. Algorithms were tested by simulation on simple objects and on a model of two-mass system. The research was made in Matlab/Simulink environment.

## KEYWORDS

input shaping, command generation, autotuning, two-mass system

## 1. INTRODUCTION

The problem of avoiding oscillations is one of the most challenging tasks of contemporary automatic control systems. Every mechanical oscillation causes many unwelcome effects – the life time of clutches and gears is strongly reduced, and energy consumption of the drives is greater etc. The standard control methods like PID controllers are usually insufficient to reduce the oscillations in the systems with flexible joints. This problem is not marginal – every crane, robotic arm, conveyer or even simple drive contains flexible joint. Many of these objects can be modeled by multi-mass system (or in the simplest example by two-mass system).

The wide range of control strategies used to damp the oscillations have been developed. Digital filters in feedback, adaptive controllers or artificial neural network causes many problems in implementation, tuning or usage. One of the simplest and most effective strategy to avoid the oscillations is input shaping. The paper is based on a work presented on 16th Mechatronika 2014 Conference [Gniadek 2014].

## 2. INPUT SHAPING

### 2.1 Input shaping basics

Input shaping is a very simple algorithm, that can be used to reduce the object's oscillations. The main idea of the input shaping method is based on convolution of the baseline command with a sequence of Dirac impulses. The impulses should be applied in specified moments of time and with specified amplitude. The response of each impulse should be in antiphase to reduce each other. The main idea of input shaping is shown in figure (1) [Singhose 2011].

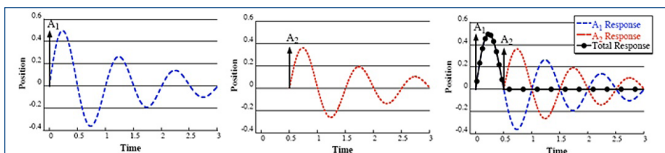


Figure 1. Input shaping – basic idea

To achieve the oscillating object response like shown in figure (1) the simple algorithm of input shaping was used. The input command was convolved with a series of two impulses. The moment of application of the second signal has to be exact in the half of primary response period ( $T_d/2$ ). This simple algorithm can be presented symbolically as shown in figure (2) [Singhose 1997].

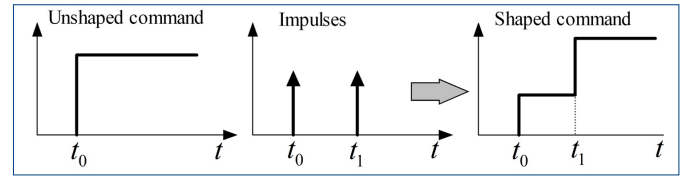


Figure 2. Correct input shaping

### 2.2 Oscillation reduction

To understand why the input shaping reduces the oscillations, some basic examples will be needed. Consider the second-order object with two imaginary poles (with not damped oscillations). This object's step response can be presented as convolution of two signals like presented in equation (1):

$$y_0(t) = \omega_n A_0 \sin(\omega_n(t-t_0)) \square(t-t_0), \quad (1)$$

where  $\omega_n$  is natural pulsation,  $A_0$  is amplitude, and  $t_0$  is the start time. The response of the system for a sequence of impulses can be described by [Sorensen 2008]:

$$y_{ss}(t) = \omega_n \sqrt{C^2 + S^2} \sin(\omega_n t - \Psi), \quad (2)$$

where

$$C = \sum_{i=1}^n A_i \cos(\omega_n t_i), \quad S = \sum_{i=1}^n A_i \sin(\omega_n t_i), \quad \Psi = a \tan\left(\frac{S}{C}\right). \quad (3)$$

According to equations (1) and (2) the proper moments of time and amplitudes of impulses are required to reduce the oscillations. The equations to calculate the times and amplitudes are different for different algorithms of input shaping. The paper is based on the simplest algorithm of command generation. This choice is supplied by the statement, that the shaper does not have to be very robust – if the object parameters are changed the algorithm can be simply executed. For the selected type of input shaper the impulses amplitudes ( $A_i$ ) and times ( $t_i$ ) of applying can be calculated from equation [Brock 2014a]:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & K \\ 1+K & 1+K \\ 0 & 0.5T_d \end{bmatrix}, \quad (4)$$

where

$$K = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right), \quad (5)$$

$T_d$  is the period of oscillation, that has to be damped, and  $\zeta$  is the damping coefficient for the selected resonant frequency.

This algorithm is effective only if the resonant frequency is known and constant. Otherwise, the usage of more complicated structure of input shaping is considerable. Usage of more robust algorithms will increase the rise time and settling time.

Algorithm presented in equations (4) and (5) is sufficient if the object is not changing the parameters. If the mass, moments of inertia etc. are changing, the resonant frequency is changed respectively. The presented algorithm is also simple to supply the real system – usually

the system has a couple of resonant frequencies (object is 4th or higher order). The detection of multiple convolved and damped sine waves is more complicated. The algorithms presented in this paper are one of possible, automatic solutions to solve this task.

### 2.3 Multiple frequencies damping

As in the previous chapter was mentioned, usually the object's output is a sum of multiple sine waves. To damp every frequency that might cause problems, the algorithm has to be modified. The method presented in [Singhose 1997] shows that simple shaper can be used to damp a single oscillation frequency. Multiple shapers connected in series can damp multiple frequency. To simplify the object structure, the equivalent of multiple simple shapers can be designed. The composite shaper is constructed due to a convolution of simple shapers. Exemplary, having two robust shapers with parameters  $A_1, t_1$  and  $A_2, t_2$ :

$$\begin{bmatrix} A_1 \\ t_1 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0 & 1 & 2 \end{bmatrix},$$

$$\begin{bmatrix} A_2 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0 & 3 & 6 \end{bmatrix},$$

the convolved shaper will be equal to:

$$\begin{bmatrix} A_c \\ t_c \end{bmatrix} = \begin{bmatrix} \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}.$$

This basic dependency simplifies the algorithms of autotuning because unifies the structure of system. Only a single shaper is always present, the complexity is changed, but the structure remains still.

## 3. AUTOTUNING ALGORITHM BASED ON OBJECT IDENTIFICATION

### 3.1 Introduction

As was mentioned in the previous chapter the information about the natural oscillation frequencies and their damping is required to tune the input shaper. The most natural approach in extracting this information is the object identification. The location of transfer function poles is determining those parameters. The natural pulsation is equal to the distance between the selected pole of transfer function and zero point on complex plane. The damping factor is equal to the cosine of angle between negative real semi-axis and line segment between zero point and specified pole.

If the root is located on the real axis the damping coefficient is equal to 1. With this parameter the root will not cause any oscillations. If the pole is on the imaginary axis the oscillations will not be damped.

Of course, in practical situation, the exact transfer function is unknown. For this reason the object has to be identified. The algorithm of identification can be chosen from the wide range of well-known and described algorithms. This paper is based on one of the simplest algorithms – the Least-Square method. The basis of this algorithm is presented in multiple papers about objects parameters identification.

If all parameters of oscillation are calculated all the demanded information to tune the shaper is available. This method will work properly only for object with slight nonlinearities.

### 3.2 Algorithm description

The autotuning of input shaper using the identification method requires following steps:

#### 1) Object identification

The object input and output are supplied to the least-square algorithm. The model of plant is linearized and transfer function is extracted. If the order of transfer function is lower than 2, the input shaper is not needed and algorithm is terminated.

#### 2) Poles calculation

When the transfer function is known, the denominator's roots are calculated. If there are no poles with imaginary part different from zero, the algorithm is terminated and input shaping is not needed.

#### 3) Poles selection

The roots of characteristic equation are being checked for stability. If any of roots has positive real part, the object is unstable and additional control method is required. All of complex roots are selected – one of conjugated pair of complex roots is sufficient to construct the shaper (the second one contains redundant information). Optionally only some of the roots can be used in the next step – if the damping factor associated with the pole is close to one the pole can be omitted. This operation will positively influence on the control time and will insignificantly negatively influence on the control quality.

#### 4) Simple shapers designing

Every of  $n$  selected roots is a base for one simple input shaper. The data needed for equation (4) are calculated:

$$\text{for } k = 0..n \quad p_k = X + Yi \quad (6)$$

where  $p_k$  is one of the poles,  $X$  is the real part of the  $p_i$  and  $Y$  is the complex part of  $p_k$ . The  $T_d$  is equal to:

$$T_{d_k} = \frac{2\pi}{\sqrt{X^2 + Y^2}} \quad (7)$$

and damping rate is equal to:

$$\zeta_k = \left| \frac{X}{\sqrt{X^2 + Y^2}} \right|. \quad (8)$$

The equation (8) is proper only if  $X \leq 0$ . This condition was checked in step 3.

#### 5) Shapers convolution

The simple shapers has to be convolved in a loop into one complicated shaper. The convolution strategy is described in part 2.3. The simple shapers are designed to prevent oscillations for all oscillation amplitudes selected in previous points of algorithm. The primary formal is summing the amplitudes. If it is equal to 1 the algorithm was prepared properly.

## 4. AUTOTUNING ALGORITHM BASED ON FOURIER TRANSFORM

### 4.1 Introduction

In various practical situations the object identification is very problematic or sometimes even impossible. If the object is not linear the calculations will be invalid for many operating points. To solve this problem the second approach based on the Fourier Transform is presented.

The Discrete Fourier Transform (DFT) converts the signal given as a finite list of samples to a finite combination of complex sinusoids, ordered by their frequencies. This property enables to extract the information about signal frequencies if sampling frequency is known. The main problem of this method is to get the damping factor in more complicated signals.

### 4.2 Algorithm description

#### 1) System preparation

The object input is switched into a step function. The final value of the step should be limited to a safe value, but must not be too low – the oscillations of output have to be easy to measure. The other input signals (ex. Pseudo random signal) may be used

#### 2) System test

The prepared system should be tested. Fixed sampling frequency of the output is required. The test should not be shorter than 10 periods of natural oscillations with lowest frequency, that should be damped.

#### 3) Fourier transform

The output should be transformed using Fourier transform. The oscillations frequencies are saved as 2-by- $n$  matrix, where first row contains frequencies

and second contains the magnitude of a single frequency. The anti-aliasing filter should be used to avoid the duplication of single information

4) Frequencies filtration

DFT on a digital signal will contain many unwanted information, that should be filtered. Two types of filtration are available – the first one gives as a result N peak frequencies with maximal magnitude (from step 3). The second filtration type is automatically removing the frequencies with magnitude lower than a parameter A and allows researcher to define width of a single impulse from DFT. This filtration allows shaper to work faster and with higher accuracy.

5) Natural oscillation periods calculation

The oscillation periods are demanded to the equation (3). All filtered frequencies are calculated according to equation:

$$\omega_{d_i} = 2\pi f_i \tag{9}$$

If the output signal is a combination of more than one harmonic signal, the damping ratio cannot be easily calculated. In this case each simple input shaper's amplitudes are arbitrary set to [0.5 0.5].

6) Shapers convolution

The simple shapers has to be convolved in a loop into one complicated shaper (like in part 3.2–5). Finally the shaper is being tested. The plant is changed to the version before point 1.

5. AUTOTUNING ALGORITHM VERIFICATION

The theory presented in previous chapters has to be verified. To verify the algorithms the test on objects has to be executed. The objects will be stimulated with step function. Each test will be presented on a chart. The chart will contain 3 courses – the unshaped, shaped with shaper tuned using object identification and shaped using FFT. The verification will be done using Matlab/Simulink environment. The Simulink model is presented in figure (3).

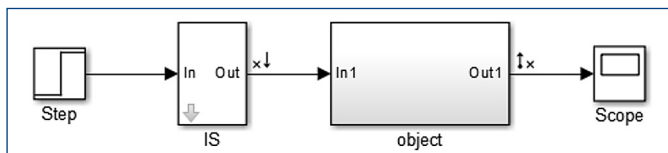


Figure 3. Verification model – structure

Test will be executed on 3 different objects – two simple transfer functions and a two mass system model. All the results are presented in parts 5.1–5.3.

5.1 Simple second order transfer function.

The first test will be provided on the simplest possible oscillating object – the second order oscillating object with damping. The object transfer function is equal to:

$$G_{obj}(s) = \frac{1}{s^2 + 0.2s + 0.3}$$

The roots are located in points  $0.1 \pm 0.54i$ . The shaper was auto-tuned with both methods. The results are presented in figure (4):

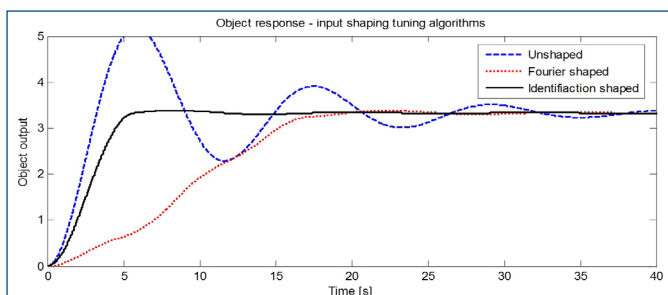


Figure 4. Simple object response

The figure (4) proves that both algorithms are working. The oscillations are damped to 0. The algorithm based on object identification works with a great accuracy. The object response is fast, the oscillations are damped. The shaper tuned with DFT works, but the response time is bad. The dynamics is so low because of the shaper – the autotuner has found 3 additional frequencies, that are not existing in the object. The baseline signal is convolved with 8 impulses what influences to the dynamics.

5.2 Simple transfer function, fourth order.

The second test was executed on the fourth order object. To simplify the interpretation of results the fourth order object was created due to two second order objects connected in series. The transfer function of this object was equal to

$$G_{obj}(s) = \frac{1}{s^2 + 0.2s + 0.3} \cdot \frac{1}{s^2 + 0.25s + 0.4}$$

Other conditions are the same as in the previous point. The preliminary tests were made using the simple algorithm of root filtration. The results are presented in figure (5).

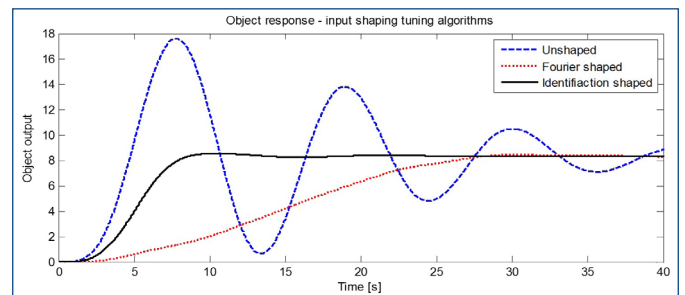


Figure 5. 4th order object response

The shaper based on object identification works, as in previous example, with high accuracy, the oscillations are damped and dynamics is very good. The Fourier shaping has met the same problem like in previous example. To eliminate it's occurrence advanced selection of roots was made. The results are presented in figure (6).

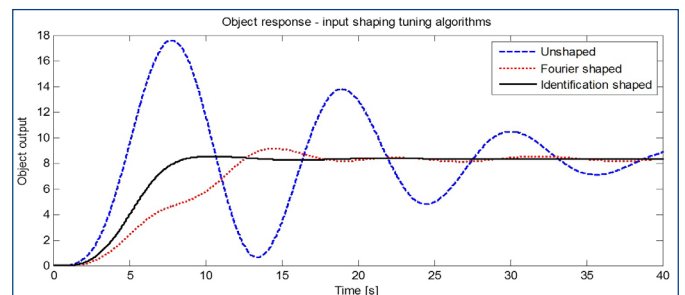


Figure 6. 4th order object response – advanced roots filtration

The shaper based on object identification was not changed. The FFT algorithm had more complicated algorithm of poles filtration. The dynamics of this object has significantly increased. The results are much better. The oscillations are visible, but are significantly reduced, but are visible because the damping coefficient is omitted (every impulse has the same amplitude).

5.3 The two-mass system

The tests provided on the simple objects given as transfer function are good for the primary tests. The real verification has to be executed on more complicated object. The most common example of oscillating objects is two-mass system. The object is built for two masses connected with the elastic shaft. The speed of the first mass is known, but no information about the second mass is given. The idea of two-mass system is described in many papers. The testing model is described in

[Luczak 2014]. The general structure of two-mass system is presented in figure (7):

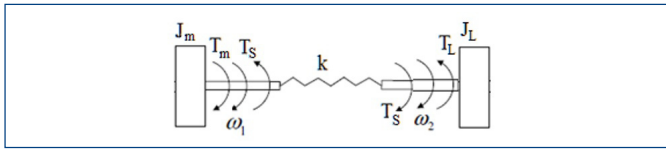


Figure 7. Two-mass system

where  $J_m$  is moment of inertia of the first mass (drive motor),  $J_L$  is moment of inertia of the second mass (load),  $T_m$ ,  $T_L$  and  $T_S$  are the torques transmitted through the shaft – the torque of motor, load and springiness respectively,  $k$  is the springiness coefficient and  $\omega_1$  and  $\omega_2$  are the speeds of both masses.

The object response (speed of load mass) is presented in figure (8).

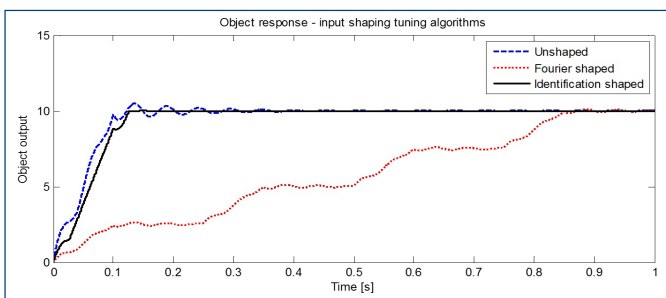


Figure 8. Two-mass system response

The results of the test for identification tuning method are good. The dynamics of the object with shaper is maintained, the oscillations are reduced to 0. The results for Fourier tuning are bad. The oscillations are not damped and dynamics is strongly limited. The detected period of oscillation is different from the actual. Advanced selection of roots will require the expert's knowledge what is negation of the autotuning idea.

## 6. CONCLUSION

Two algorithms of input shaping autotuning were designed, prepared and tested. The theoretical bases for the algorithms were different. Both algorithms are working but the efficiency of their work is various.

The algorithm based on object identification is working, according to the theory, with good accuracy. The oscillations are damped, the dynamics is high. The tuned shaper ensure proper work of whole system. The amplitudes and delays calculated by the algorithm are nearly same as counted using classical methods of input shaper designing.

The algorithm based on the FFT is not working with high accuracy. The reduction of oscillation is significant, but the dynamics of the object is low. The algorithm needs plenty of additional operations provided with expert's knowledge for dully satisfying work. Sometimes this approach is the only possible. Because of this reason it is very important that the algorithm is generally working.

The experiments have been conducted also for higher order objects. The Input Shaper tuned using identification was working with very good accuracy for all of tested objects (form 2nd up to 100th order). According to the mathematical basis described in [Singhose and Seering 2011] the shaper is reducing the oscillations to 0 as long as all the plant is linear and all the parameters are known. If there are some non-linearities in the control system the shaper will reduce, but not eliminate the oscillations.

The shaping structure may also be connected with controllers using feedback. The strategy is presented in [Gniadek 2015] and [Brock 2014b]. Other papers are also presenting the possibilities of industrial applications of input shaping.

The main target of the paper was achieved – auto tuner was designed, tested and works properly. The oscillations are damped with high efficiency.

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## CONTACTS:

MSc. Eng. Maciej Gniadek  
Poznan University of Technology,  
Institute of Control and Information Engineering  
Piotrowo 3a, 61-138 Poznan, Poland  
e-mail: maciej.gniadek@amica.com.pl  
www.put.edu.pl

Amica Wronki S.A.  
ul. Mickiewicza 52, 64-510 Wronki, Poland

Assoc. Prof. Eng. Stefan Brock  
Poznan University of Technology,  
Institute of Control and Information Engineering  
Piotrowo 3a, 61-138 Poznan, Poland  
e-mail: stefan.brock@put.poznan.pl  
www.put.edu.pl